

Housing Boom, Mortgage Default and Agency Friction^{*}

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Abstract

The housing prices and the mortgage debt witnessed faster growth than GDP in the run-up of the Great Recession. I document a mortgage market puzzle during the boom period: (1) the mortgage risk measured by the *ex post* delinquency increased, but (2) the mortgage spread decreased. The default premium alone cannot explain the decreasing mortgage spread in the boom episode. I develop a dynamic general equilibrium model of the housing and the mortgage markets with borrowers, depositors, and intermediaries to explain the empirical fact. The model features the tightness of the lending condition and the mortgage risk as the aggregate shocks, which generate the time-varying liquidity and default premiums in the mortgage spread. I quantify the contribution of the aggregate risks to the boom-bust dynamics before and after the Great Recession. A plausible size of the income shock alone is insufficient to generate the observed movement in the mortgage spread. The model shows that the lending relaxation that eases the leverage constraint of an intermediary leads to the increasing mortgage credit and the decreasing mortgage spread in the boom period. The lending condition shock generates pro-cyclical leverage of intermediaries that amplifies the aggregate shocks in the boom-bust dynamics.

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1 Introduction

The housing and mortgage markets are central to the discussion on the causes and consequences of the Great Recession. From 2001 to 2007, the household mortgage debt saw a cumulative increase of more than 100%, compared to a 40% increase of GDP.¹ Empirical evidence shows that the shift from the agency loans funded by the Government Sponsored Enterprises to the non-agency loans fueled the credit boom (Levitin and Wachter, 2011). While the non-agency loans were more affordable to mortgage borrowers, they turned out to be riskier than the agency counterparts. Loans originated near the end of the boom episode performed much worse in terms of delinquency, suggesting an increase in the mortgage risk before the Great Recession (Demyanyk and Van Hemert, 2011). At the same time, the mortgage spread decreased, with the 30-year fixed rate mortgage spread and the excess return of the mortgage-backed securities decreased by about 60 bps and 500 bps respectively.² The paper addresses the mortgage market puzzle why the mortgage spread decreased in the presence of the increasing mortgage risk in the run-up of the Great Recession.

I build a general equilibrium model of borrowers, depositors and financial intermediaries in a dynamic economy. The empirical evidence suggests that, besides the mortgage risk component, another time-varying factors decreased the mortgage spread in the boom episode. I introduce and calibrate the mortgage risk and the lending condition as the aggregate shocks that capture the time-varying factors related to the credit demand and supply respectively. An intermediary issues a risk-free bond to the depositors and a long-term defaultable mortgage to the borrowers for housing purchase, with the borrowing constrained by a fraction of the housing value as the collateral. The intermediaries have two features. First, intermediaries are lending constrained, facing an endogenous cap on the leverage due to the agency friction. Second, intermediaries are risk-averse and issue defaultable loans, with the book equity owned by the depositors. These two features contribute to the liquidity and default premiums in the mortgage spread.

I distinguish the lending relaxation that receives less attention in the literature from the borrowing relaxation. The endogenous constraints on the borrowing and the lending jointly impact the mortgage credit and the mortgage spread in the model. The mortgage credit depends on the shadow value of the housing collateral which increases the borrowing limit, and the shadow cost of the mortgage which reflects the likelihood of future default and decreases the borrowing limit. The mortgage spread summarizes the tightness of both the borrowing and lending constraints. I show that two constraints have different general equilibrium impacts. While the mortgage spread increases with the borrowing relaxation, it decreases with the lending relaxation.

The intermediaries play the key role to amplify the aggregate shocks through the balance sheet channel. Due to a higher marginal value of the net worth in the boom than in the bust episode, the leverage of an intermediary is pro-cyclical and the mortgage spread is counter-cyclical. The model generates a direct serial correlation between the current and the future leverage of an intermediary, meaning that the current choice made by an intermediary depends on the future lending tightness. An intermediary finds it optimal to extend the duration of a positive deviation of the leverage, because by doing so, the marginal value of the net worth and an intermediary's present discounted value increase.

Using the quantitative framework, I simulate the boom-bust episodes and quantify the contribution of the income, the mortgage risk and the lending condition as the aggregate risks to the mortgage market dynamics. I find that the income channel alone cannot generate strong movement in the mortgage spread

¹ For the US household mortgage balance, see Z.1 Table from Board of Governors [LA153165105.Q]. For the nominal GDP, see NIPA tables from BEA [A191RC].

² See Section 2 for evidence and details.

under plausible sizes of shocks. The lending condition explains the downward movement of the excess return in the boom episode, while the mortgage risk accounts for the spike of the mortgage spread in the bust episode. I decompose the mortgage spread into a liquidity premium and a default premium. The former measures intermediary’s ability to arbitrage away the excess return, while the latter arises due to risk aversion of intermediaries and the presence of mortgage default. I show that the default premium increased and responded to the increasing mortgage risk in the boom, but was quantitatively outweighed by the decreasing liquidity premium. The lesson is that the macro factors about the credit supply was time-varying in the credit boom and are essential to explain the time series behaviors of the mortgage spread before the Great Recession.

1.1 Related Literature

The paper contributes to several threads of the literature. Since the Great Recession, the empirical literature that links housing prices, mortgage default, and economic activities has been growing. Evidence shows that the housing boom in the run-up of the Great Recession was attributed to the excess mortgage credit expansion and the underpriced credit risks responsible for the subsequent surge of the mortgage default.³ The paper complements the empirical work by analyzing the transmission mechanism and evaluating the macroeconomic impacts in a general equilibrium model with endogenous housing prices and interest rates.

The paper builds on the macro literature on housing and mortgage.⁴ I explore the causes and consequences of the housing boom and quantify the contribution of different factors. The model with aggregate risks shares the general equilibrium implication in quantitative work with [Elenyev, Landvoigt and Van Nieuwerburgh \(2016\)](#), [Favilukis et al. \(2017\)](#), [Landvoigt \(2017\)](#) and [Greenwald \(2017\)](#). Different from the previous work, the model features an endogenous cap on the leverage managed by long-lived and risk-averse intermediaries.

Most studies on the role of credit constraints in housing literature focus on the financial friction of credit demand. Relaxing the maximum loan-to-value ratio constitutes the main experiment in recent studies ([Iacoviello and Pavan, 2013](#); [Landvoigt et al., 2015](#); [Favilukis et al., 2017](#); [Kaplan et al., 2017](#)), while liberalization of payment-to-income ratio has recently been brought to discussion in literature ([Corbae and Quintin, 2015](#); [Greenwald, 2017](#)). The problem of credit demand relaxation lies in the lack of micro evidence and foundation of the constraint shift ([Kaplan et al., 2017](#)).

This paper studies the joint impact of the borrowing and the lending constraints. Of particular relevance, [Justiniano et al. \(2017\)](#) build a model with a borrowing constraint and an exogenous lending limit. They argue that the lending relaxation instead of the borrowing relaxation is consistent with the decreasing mortgage rates and the stable mortgage-to-real estate ratio before the Great Recession. By endogenizing the maximum lending limit, my model rationalizes the lending relaxation as an optimal response of an intermediary to the state of economy and supports the credit supply view on the constraint liberalization in the boom episode.

This paper is related to the macroeconomic implication of intermediation frictions pioneered by [Kiyotaki and Moore \(1997\)](#), [Carlstrom and Fuerst \(1997\)](#) and [Bernanke et al. \(1999\)](#). I base the analysis on the recent framework developed in [Gertler and Kiyotaki \(2010, 2015\)](#) and [Gertler and Karadi \(2011\)](#) where the

³ [Mian and Sufi \(2009\)](#) finds securitization facilitated fast subprime credit expansion in areas with low income growth. Unsustainable growth of mortgage credit leads to the surge of default rate in the Great Recession. [Levitin and Wachter \(2011\)](#) show that the housing bubble is a supply-side rather than a demand-side phenomenon. The lending boom is associated with the shift of the market from the regulated Agency securitization of traditional fixed-rate mortgages to unregulated private-label securitization of nontraditional adjustable-rate mortgages. The originate-to-distribute model of lending worsens information asymmetry between securitizers and investors ([Stein, 2002](#); [Rajan et al., 2015](#); [Levitin et al., 2019](#)), and distorts intermediary’s incentives of screening and mortgage origination ([Keys et al., 2010](#); [Purnanandam, 2010](#); [Keys et al., 2012](#)).

⁴ For recent overviews on housing and macroeconomics, see e.g. [Davis and Van Nieuwerburgh \(2015\)](#), [Guerrieri and Uhlig \(2016\)](#), [Piazzesi and Schneider \(2016\)](#).

limited enforcement of debt contracts results in the constraint on an intermediary’s leverage. My model focuses on the news effect of future leverage and future lending tightness that determines current marginal valuation on net worth. Related effects in intermediary asset pricing are discussed in [Bocola \(2016\)](#), [He and Krishnamurthy \(2013\)](#) and [Brunnermeier and Sannikov \(2014\)](#).⁵ The novel insight is that the future lending tightness affects current valuation on net worth and asset prices.

Finally, I build on the mortgage default literature and bring new insight to the analysis.⁶ Mortgage default in my model is a put option on housing prices, exercised by the borrowers. In the same spirit as [Jeske *et al.* \(2013\)](#) and [Landvoigt \(2016\)](#), my model uses idiosyncratic housing price risk to rationalize counter-cyclical mortgage default and to motivate partial default from an aggregate perspective. The mortgage risk is defined as the standardized dispersion of the housing price risk who has a Pareto distribution.⁷ The setup features tractable characterization in a model with mortgage default, financial intermediaries and aggregate risks, allowing me to solve most variables at the steady state quasi-analytically.

Overview. The rest of the paper is organized as follows. In Section 2, I summarize the key facts in the mortgage market in the run-up of the Great Recession. In Section 3, I outline the model and the equilibrium concept. Section 4 simplifies the model to illustrate the key mechanism. Section 5 describes the calibration. Section 6 simulates the boom-bust episodes and conducts counterfactual experiments. Section 7 concludes. Data and additional results can be found in the appendices.

2 Overview of US Mortgage Finance Towards the Great Recession

The section summarizes the key facts about the residential mortgage finance towards the Great Recession. The period 2001-2007 witnessed faster growth of mortgage debt than the income. The mortgage-to-disposable income ratio grew by more than 25 percentage points to reach 70% right before the Great Recession (Figure 1a).⁸ Instead of depositors, intermediaries that issue mortgages are the credit suppliers to the households, essential to the pricing of mortgage risk. My objective is to present two facts in the boom episode: increasing mortgage risk and decreasing mortgage spread. Details on data sources and definitions are in Appendix A.

2.1 Increasing Mortgage Risk

2.1.1 Shift towards non-Agency and Adjustable Rate Mortgages

Mortgage finance from 2001 to 2007 shifted from the agency mortgage backed securities (MBS) funded by the government towards non-agency MBS by the private entities. By 2001, the agency loans had dominated the MBS market, covering more than 80% of the securitized loans. The share of the agency MBS decreased

⁵ In an extended framework of intermediation with productive capital and unproductive government bond, [Bocola \(2016\)](#) focuses on the contagion effect of sovereign credit risk that leads to slower capital accumulation. Expecting higher sovereign risk lowers the marginal valuation of assets and thus reduces the incentives to hold risky asset due to precautionary motive. [He and Krishnamurthy \(2013\)](#) and [Brunnermeier and Sannikov \(2014\)](#) link intermediaries’ financing friction to return premium, but explores the effect of an equity rather than a debt constraint on the nonlinearity of return premium in crises.

⁶ The macro literature on mortgage finance considers either strategic or non-strategic default, depending on whether there is default cost beyond house foreclosure. For papers of strategic default, see [Chen *et al.* \(2013\)](#); [Corbae and Quintin \(2015\)](#); [Chatterjee and Eyigungor \(2015\)](#); [Kaplan *et al.* \(2017\)](#). For papers of non-strategic default, see [Jeske *et al.* \(2013\)](#); [Landvoigt \(2016\)](#); [Elenev *et al.* \(2016\)](#). The paper follows the non-strategic approach to model mortgage default.

⁷ The standardized dispersion as the risk measure in economics dates back to [Atkinson \(1970\)](#) in which to measure inequality across different classes of distributions.

⁸ The mortgage-to-real estate ratio since 2001 remained stable at 38% until 2008 (Figure 1b). It increased by 20% in 2 years, and reached the historical peak, 56%, in late 2009. The sudden increase is mainly driven by the plummet of real housing price during the crisis episode, which plummeted by more than 30% from the peak in 2006 (Figure 2).

by about 15% and 20% relative to the total residential mortgage outstanding and the total MBS outstanding, respectively (Figure 3). The share of loans securitized in the period was stable at 60%.⁹ Hence, the MBS market didn't witness change in the extent of risk transfer through securitization, but drastic composition change from agency to the non-agency loans.

On the other hand, the mortgage product composition shifted from the fixed rate mortgages (FRM) towards the adjustable rate mortgages (ARM). The evidence is based on a database representative of the non-agency securitized loan market.¹⁰ Prior to 2003, ARM loans accounted to almost half of the private-label securitized loans by the dollar volume. From 2004, ARM loans dominated the market, covering more than 80% at the historical peak by the dollar volume (Tables 1 and 2). Many hybrid loans through the private-label conduit were innovated with the features of both ARM and FRM loans, and aimed to attract borrowers by reducing the payment size at the initial periods of a loan term (Levitin and Wachter, 2011). These loans featured balloon payment, interest-only periods, negative amortization.¹¹ As there were no well calibrated risk models of those hybrid loans due to the short history of loan performance, the wild and unregulated credit growth accumulated mortgage risk during the housing boom.

2.1.2 Surge of Delinquency and Foreclosure

The surge of delinquency and foreclosure at the end of housing boom showed severe credit loss due to the accumulating credit risks in the run-up of the Great Recession (Figure 4a). The foreclosure rate increased by six folds, going up from 0.6% before the crisis to the historically high 3.6% in late 2011. The delinquency rate peaked earlier in late 2009 at 8.3% and 12.5% for loans past due more than 90 days and more than 30 days, respectively. Figure 4b shows that the non-agency loans whose origination was closer to the bust episode had worse loan performance in the Great Recession, suggesting an increase in the mortgage risk. The share of active loans at the end of 2007 that became delinquent in 2008 were 8% higher for loans originated during 2005-2007 than those during 2001-2004, a 35% increase in delinquency share for the post-2005 vintages.

2.1.3 Shift towards Risky Borrower and Loan Characteristics

Financial innovation turned out to worsen the quality of the mortgage pools by facilitating credit access more to the subprime borrowers who would have been denied by traditional mortgages. The average FICO credit score of non-agency securitized adjustable rate mortgages decreased from 710 in early 2003 to 680 in the mid 2006 (Figure 5a), while the loan-to-value ratio (LTV) witnessed an increase by more than 10 percentage points in the same period, reaching an average of 78% in the mid 2006 (Figure 5b). As ARM loans dominated from 2004, decreasing FICO score and increasing LTV ratio reflected worsening quality of the average borrower and increasing credit risks before the financial crisis.

The loosening underwriting standard at mortgage origination provides another piece of evidence for increasing credit risks. The share of loans with full documentation at origination kept decreasing from 60%

⁹ The securitized share is defined as the share of mortgage debt not held by depository institutions, but held by federal agencies, mortgage pool and trusts, or individuals and others through mortgage backed securities. The mortgages not held by depository institutions are interpreted as the assets held not by mortgage origination process, but by securitized loans that are tradable. Data is available at Board of Governors of the Federal Reserve System, 1.54 Table.

¹⁰ The data set is generated by the Wells Fargo Bank who serves as the trustee for the non-agency MBS. While these loans cover approximately 10% of the non-agency market share, the Bank was listed as the top retail mortgage lender and servicer nationwide (Inside Mortgage Finance, 2012). See Appendix for details.

¹¹ ARM loans between 2001 and 2007 witnessed an average annual growth rate of 120% by the dollar volume, while the growth rate of FRM loans was only 98%. Balloon loans have a large share of principal repaid at the end of loan term. Interest-only loans usually only need to repay interest portion for the first few years. Negative amortization loans have growing mortgage balance over time. These features are not mutually exclusive.

in 2001 to 40% in 2007 (Figure 5c). Due to increasing share of low documentation (stated and limited) with stated or non-verifiable information, information asymmetry was exacerbated and evaluation of credit risk and future cash flow became more difficult.¹²

2.1.4 Increasing Bank Leverage

Leverage ratios of financial intermediaries provide additional evidence of increasing risks from the supply side of credit. Evidence from Compustat microdata on the banks in US shows that the banking sector as a whole experienced an increase in the leverage; the total asset-to-total equity ratio increased from 15.6 in 2004q1 to 19.9 in 2008q4 (Figure 6a). Similar evidence on increasing macro leverage is found in the sample of US commercial banks and is consistent with the literature (Adrian and Shin, 2009).¹³ From a micro perspective, the median leverage ratio slightly trended downward from 11.3 in 2001q1 to 10.8 in 2007q4.(Figure 6b). However, the unweighted ratio excludes off-balance sheet consideration. The Tier 1 leverage ratio which assigns risk weight to both on- and off-balance sheet items would be a better indicator of true financial health.¹⁴ While they co-moved most of the time, the median Tier 1 leverage ratio and the unweighted leverage ratio went in the opposite direction from 2004 to 2007. The former increased from 9 to 10, while the latter decreased from 11.4 to 10.7. The opposite implication of two ratios indicates that banks leveraged up by taking more risks through off-balance-sheet expansion before the financial crisis.

2.2 Decreasing Mortgage Spread

The credit expansion together with increasing mortgage risks was accompanied by a decreasing mortgage spread. I first present robust evidence on the mortgage spread of agency loans in the primary and secondary mortgage markets. In the primary mortgage market, the 30-year fixed rate mortgage rate over 10-year treasury rate decreased by 60 bps from 2.1% in 2000 to 1.5% in 2007 and followed by a surge to 2.6% in 2008q3 (Figure 7a).¹⁵ The funding cost was reduced by almost 30% in the boom episode.

In the secondary mortgage market, the Barclays MBS Index tracks the performance of the agency mortgage pass-through securities. From 2001 to 2007, the total return of the MBS Index over 1-year treasury rate decreased by more than 500 bps (Figure 7b).¹⁶ It reflected investors' valuation on mortgage credit risk had decreased before the crisis. Alternatively, the option adjusted spread (OAS) of Barclays MBS Index is based on net-present-value and simulation models used in the mortgage industry to account for the embedded options (prepayment and refinance) of a mortgage contract. OAS decreased by 70 bps from 1.1% in 2000 to 0.4% in 2007 followed by a surge to 1.6% in 2009 (Figure 7b). The trend of the risk prices in the secondary market was consistent with the pattern in the primary mortgage market.

¹² At mortgage origination, a lender may demand different level of information on income and employment (W-2, pay stub, tax return etc), and asset and debt (bank account statement, market value of stock and bond, car loan, student loan etc). The documentation types can be classified into three cases based on information verifiability and creditworthiness: full, limited and stated. See p.110 in [Financial Crisis Inquiry Report \(2011\)](#) for details. The evidence is consistent with [Keys et al. \(2009\)](#).

¹³ The macro leverage of US commercial banks is based on the aggregate balance sheet data from Federal Financial Institutions Examination Council. The series of total asset and total equity are first log-detrended for stationarity before taking the ratios.

¹⁴ The Tier 1 leverage ratio is the inverse of the risk adjusted capital ratio (capital adequacy ratio) which is defined as the tier 1 equity as a% of the risk weighted assets.

¹⁵ The 30-year fixed rate mortgage spread is defined as the difference between US 30-Year Fixed Rate Mortgage Rate and 10-Year Constant maturity Treasury Rate. Because the duration of a 30-Year fixed rate mortgage is about 13 years (based on a real mortgage rate 3.3%), 10-year treasury rate is the closest choice within the class of government risk-free bonds to minimize the impact of term structure. I choose 10-year treasury rate to approximate the duration of the repayment cash flow. See [Lehnert et al. \(2008\)](#) and [Pavlov and Wachter \(2009\)](#) for the implication of duration-matched spread in the mortgage market.

¹⁶ The total return spread is defined as the MBS Index total return over the 1-Year Constant Maturity Treasury Rate.

The decreasing mortgage spread is similarly found in the ARM pools of non-agency MBS (Figure 5d). The gross margin of ARM loans showed a decreasing trend from 2001 to 2006, going down by more than 200 bps from 2001q1 to 2004q3 but slightly moving up by 120 bps from 2006q3. The trend of the non-agency fixed-rate mortgage rate over the 10-year treasury rate showed mixed results. It first decreased by 200 bps from 2001q1 to 2003q3, followed by an increase of 200 bps to 4.3% in 2006q3.¹⁷

2.3 Explaining Mortgage Credit Boom: Borrowing vs. Lending Relaxation

It sounds puzzling in the credit boom that the increasing mortgage risk was accompanied by the decreasing mortgage spread. The view of borrowing relaxation may explain the exponential growth of mortgage credit, but it is inconsistent with the stable mortgage-to-real estate ratio (Figure 1b). More importantly, it cannot rationalize the decreasing mortgage spread. Relaxing borrowing constraints turns out to increase the mortgage spread either by extending mortgage credit to the marginal or subprime borrowers through the extensive margin (Mian and Sufi, 2009, 2016), or by extending more credit to the prime borrowers through the intensive margin (Foote *et al.*, 2016; Adelino *et al.*, 2016; Albanesi *et al.*, 2017).

The evolution of the mortgage spread suggests that, besides the mortgage risk component, other factors in the mortgage spread were time-varying and drove down mortgage spread in the mortgage credit boom. In contrast to the borrowing relaxation, lending relaxation is consistent with the decreasing mortgage spread in the run-up of the Great Recession (Mayer *et al.*, 2009; Levitin and Wachter, 2011; Demyanyk and Van Hemert, 2011; Justiniano *et al.*, 2017). Motivated by the empirical evidence, I introduce mortgage risk and lending condition to the model as part of the aggregate risks of the economy in Section 3. Beyond their literal meanings, the mortgage risk and the lending condition compactly summarize the time-varying macro conditions of the credit demand and supply, respectively.

3 Model

3.1 Environment

The model involves two types of household, borrower (B) and depositor (D). The households are labeled according to their equilibrium behaviors, due to the difference in time preference $0 < \beta_B < \beta_D < 1$. The financial intermediary owned by the depositor household plays the role of maturity transformation and risk allocation by channeling the fund from the depositors to the borrowers. There is a fixed amount of divisible house Lucas tree (called *house* and normalized to 1) in the economy to provide one-to-one housing service.

There are three sources of friction. First, the borrower household faces a borrowing constraint that restricts the demand of mortgage credit. Second, the depositor household has limited participation in the risky asset investment, but has to deposit fund in the financial intermediary that purchases the mortgage backed security. Third, the intermediary faces an incentive constraint due to moral hazard, which limits the supply of mortgage credit. The market structure of the economy is illustrated in Figure 8.

¹⁷ An MBS investor receives the principal and the interest payment of the underlying mortgages, minus the commission of the servicer and the trustee as a percentage of repayment. The data on the PLMBS return spread are not available in general (because only agency MBS are traded in the To-Be-Announced (TBA) Market. See SIFMA TBA Market Fact Sheet available at <https://www.sifma.org/wp-content/uploads/2011/03/SIFMA-TBA-Fact-Sheet.pdf>). The mean mortgage spread of the underlying mortgages will be a good proxy for the return spread of a mortgage pass-through security.

3.2 Borrower Household

There is a representative borrower household with unit mass of infinitely many borrowers. In the family, perfect insurance is provided against idiosyncratic risks. The borrowers value both non-durable consumption and housing service, and hold long-term defaultable mortgage debt using house as collateral. Each individual borrower is endowed with a share of house subject to an idiosyncratic house price risk at each period. The long-term mortgage is the only financial contract to finance house purchase.

3.2.1 Long-term Defaultable Mortgage

The representative borrower uses long-term defaultable mortgage to finance house purchase. At the beginning of a period, each individual borrower inherits an equal share of existing mortgage m_{t-1}^B and house h_{t-1}^B from the borrower household. A borrower can make the mortgage payment plan contingent on her idiosyncratic *iid* house price risk ω_t : whether to default on her share of the mortgage (D) or to stay current (C). Denote the binary indicators of the payment plan as $\chi_C(\omega_t)$ and $\chi_D(\omega_t)$, with $\chi_C(\omega_t) + \chi_D(\omega_t) = 1$.

After making the payment plan, the borrower household chooses the size of new borrowing l_t^B at price q_t which repays the mortgage coupon that decays geometrically at rate μ from tomorrow on. The cash flow conditional on staying current from period t on is $\{q_t, -1, -\mu, -\mu^2, \dots\} \cdot l_t^B$. Thus, the gross mortgage rate implied by the mortgage price is $R_{q,t} = q_t^{-1} + \mu$. If a borrower defaults on a mortgage, the house used to collateralize the loan is transferred by foreclosure to the lender who will instead pay the maintenance cost. The mortgage is non-recourse, so there is no other penalty except the transfer of the house in the process of foreclosure. If a borrower doesn't default, the maintenance cost $(1 - \omega_t)p_t h_{t-1}^B$ has to be paid by the borrower.

With fixed rate mortgages, the borrower household is subject to an exogenous prepayment shock, requiring that a fraction $1 - \rho$ of the mortgage remaining after the payment plan should be prepaid at the face value. Prepayment includes any exogenous reason that a borrower has to terminate the contract early, due to family migration, job change, etc. Denote $\varphi \equiv \frac{\varphi_0}{1-\mu}$ as the fraction $\varphi_0 \in (0, 1)$ of the face value $\frac{1}{1-\mu}$ of future payment per unit of mortgage. The long-term mortgage whose coupon decays geometrically doesn't distinguish principal and interest payment, so a constant share of the face value is assumed to denote the mortgage principal component.

Holding a house is risky, summarized by the random variable $\omega_t \sim F_\omega(\omega_t)$ on the support $[0, +\infty)$. $\omega_t p_t$ is the idiosyncratic housing price, while the per-unit maintenance cost $1 - \omega_t$ captures the holding risk to a borrower. When $1 - \omega_t > 0$, it is the cost necessary to support future service flow produced by the house. When $1 - \omega_t < 0$, it is the investment benefit instead. In aggregate, the expected maintenance cost matches the deterministic depreciation rate of house. Because each piece of mortgage is tied to an equal share of house in the family, the mortgage risk is fully captured by the idiosyncratic house price risk.¹⁸

3.2.2 Preference and Constraint Set

Given the payment plan of individual borrowers, the borrower household chooses the size of new mortgage and house purchase. The per-period utility function takes the form

$$u(c, h) = \frac{(c^{1-\alpha} h^\alpha)^{1-\xi} - 1}{1 - \xi} \quad (3.1)$$

¹⁸ Different from the selling risk in [Jeske et al. \(2013\)](#), the maintenance cost is paid every period until a mortgage is prepaid or goes default.

where c is non-durable consumption and h is the housing service. α is the weighting share on housing service, and ξ controls risk aversion of the household. Housing service is transformed from house stock one-to-one and equally shared by borrowers in the household. Both the consumption and the housing service are delivered at the end of a period, after the payment plan and new mortgage issuance. Given the beginning-of-period house stock h_{t-1}^B and the mortgage coupon due at the beginning of the period m_{t-1}^B , the law of motion is

$$\begin{aligned} m_t^B &= \pi_t \rho \mu m_{t-1}^B + l_t^B \\ h_t^B &= \pi_t \rho h_{t-1}^B + h_{nt}^B \end{aligned} \quad (3.2)$$

where l_t^B and h_{nt}^B are new mortgage borrowing or cash-out, and new house purchase, respectively. Denote $\pi_t = E\chi_C(\omega_t)$ is the probability of repayment. On the expenditure side, the household has to pay for the maintenance $E[\chi_C(\omega)(1 - \omega_t)]p_t h_{t-1}^B$, the mortgage coupon m_{t-1}^B plus its continuation cost $\mu q_t m_{t-1}^B$, conditional on repayment. The household also pays for the new house purchase $p_t h_{nt}^B$. On the income side, the household receives the value of remaining house $\pi_t p_t h_{t-1}^B$ and after-tax income $y_t^B - T_t^B$. Additionally, mortgage prepayment requires the face value φ per unit of prepaid mortgage. Using (3.2), the flow of fund of the borrower household is

$$\begin{aligned} c_t^B &\leq y_t^B - T_t^B + q_t m_t^B - p_t h_t^B \\ &+ \int \chi_C(\omega_t) \{ \omega_t p_t h_{t-1}^B - [(1 - \rho)\varphi + \rho(1 + \mu q_t)] m_{t-1}^B \} dF_\omega \end{aligned} \quad (3.3)$$

If $\chi_D(\omega_t) = 1$, an individual borrower neither owns her piece of house asset nor mortgage liability, which yields zero value. If $\chi_C(\omega_t) = 1$, the integrand on the RHS shows the net benefit of mortgage repayment: the benefit of house value net of maintenance minus the cost of repayment obligation.

The mortgage borrowing is limited by the financial friction. The end-of-period mortgage outstanding can be collateralized by only a fraction of the market value of the end-of-period house stock. Concretely, the borrower household is subject to the following borrowing constraint, or loan-to-value ratio (LTV) constraint.

$$\varphi m_t^B \leq \theta_t^B p_t h_t^B \quad (3.4)$$

where θ_t^B specified the maximum admissible loan-to-value ratio faced by the borrower. The household is not allowed to borrow against future labor income.

3.2.3 Optimal Payment Plan

I characterize the default decision in the section. The borrower household problem can be solved in two steps. First, given m_{t-1}^B and h_{t-1}^B , I solve for the optimal default choice which is parameterized by a threshold. Second, I solve the intertemporal decision problem by choosing consumption and mortgage borrowing.

Although $\chi_C(\omega_t)$ shows up in the law of motion and the borrowing constraint, it is only relevant in the flow-of-fund constraint conditional on m_{t-1}^B and h_{t-1}^B . The integrand in (3.3) implies the default decision is characterized by a cutoff

$$\omega_t^* = \frac{[(1 - \rho)\varphi + \rho(1 + \mu q_t)] m_{t-1}^B}{p_t h_{t-1}^B} \quad (3.5)$$

below which a borrower prefers default. The threshold ω_t^* consist of two parts. The first term which relates to prepayment is the beginning-of-period debt-to-value ratio $\frac{\varphi m_{t-1}^B}{p_t h_{t-1}^B}$. The second term which relates to repayment is the beginning-of-period loan-to-value ratio. Alternatively, it is determined by the beginning-

of-period debt-to-value ratio scaled by the relative cost of continuation vs the face value, $\frac{1+\mu q_t}{\varphi}$. The flow of fund can be written in terms of ω_t^* , m_t^B and h_t^B .

$$\begin{aligned} c_t^B \leq & y_t^B - T_t^B + q_t m_t^B - p_t h_t^B \\ & + z_1(\omega_t^*) p_t h_{t-1}^B - z_0(\omega_t^*) [(1-\rho)\varphi + \rho(1+\mu q_t)] m_{t-1}^B \end{aligned} \quad (3.6)$$

where $z_i(\omega^*) = \int_{\omega^*} \omega^i dF_\omega$. The default decision thus implies $\pi(\omega_t^*) = z_0(\omega_t^*)$. T^B is the lump-sum tax from the borrower to finance the government spending. The mortgage outstanding can be regarded as a risky asset that generates the cash flow of future repayment from the pool of loans.

Given the optimal payment plan, the payoff per unit of the risky asset consists of two parts. A fraction $\pi(\omega_t^*)$ of loans won't default and yields an expected value of $(1-\rho)\varphi + \rho$. The rest of loans will default, and yields residual house value net of maintenance cost. In sum, the payoff of the asset is defined as follows.

$$Z(\omega_t^*) = \pi(\omega_t^*) [(1-\rho)\varphi + \rho] + (1-\eta) [\bar{\omega} - z_1(\omega_t^*)] \frac{p_t h_{t-1}^B}{m_{t-1}^B} \quad (3.7)$$

where $\eta \in (0,1)$ is the dead weight loss per unit of house and summarizes inefficiency of the foreclosure technology. Denote the gross return on the risky asset by

$$R_m(S_t, S_{t+1}) = \frac{Z_{t+1} + \rho \pi_{t+1} \mu q_{t+1}}{q_t} \quad (3.8)$$

where S is the aggregate states. Alternatively, the flow-of-fund constraint (3.6) can be expressed in terms of (3.8).¹⁹

$$c_t^B + p_t h_t^B + R_{mt} q_{t-1} m_{t-1}^B \leq y_t^B - T_t^B + q_t m_t^B + [(1-\eta)\bar{\omega} + \eta z_1(\omega_t^*)] p_t h_{t-1}^B \quad (3.9)$$

3.2.4 Borrower's Problem in Recursive Form

Given the law of motion of the aggregate states, the representative borrower maximizes her utility subject to the flow-of-fund constraint (3.6), and the borrowing constraint (3.4). After choosing w^* , the recursive formulation of the representative borrower's problem is

$$\begin{aligned} V^B(m_{-1}^B, h_{-1}^B; S) = & \max_{c^B, h^B, m^B} u(c^B, h^B) + \beta_B EV^B(m^B, h^B; S') \\ s.t. \text{ (FOF)} : & c^B \leq y^B - T^B + q m^B - p h^B \\ & + z_1(\omega^*) p h_{-1}^B - \pi(\omega^*) [(1-\rho)\varphi + \rho(1+\mu q)] m_{-1}^B \\ \text{(LTV)} : & \varphi m^B \leq \theta^B p h^B \\ \text{(LOM)} : & S' = \mathbf{K}(S) \end{aligned} \quad (3.10)$$

S is the aggregate states and $\mathbf{K}(\cdot)$ is the perceived aggregate law of motion. Let the Lagrange multipliers on (FOF) and (LTV) be λ_b^B and λ_l^B , respectively. The Euler equations of mortgage demand m^B and house

¹⁹ Note that R_{mt} depends on endogenous variables h_{t-1}^B and m_{t-1}^B . (3.6) is more convenient to derive the borrower's optimality conditions, while (3.9) is more convenient to derive the steady state.

demand h^B are as follows.²⁰

$$\begin{aligned} [m^B]: q - \lambda_l^B \varphi &= E\Lambda'_B \pi(\omega^{*'})[(1 - \rho)\varphi + \rho(1 + \mu q')] \\ [h^B]: (1 - \theta^B \lambda_l^B)p &= \frac{u_h^B}{u_c^B} + E\Lambda'_B p' z_1(\omega^{*'}) \end{aligned} \quad (3.11)$$

where $\Lambda'_B = \beta_B u_c^{B'}/u_c^B$ is the stochastic discount factor of the borrower household. The terms denoted by prime stand for values in the next period. LHS and RHS of the first Euler equation are the marginal value and cost of new mortgage, respectively. Today's marginal value is the amount received q per unit of mortgage. Tomorrow's marginal cost is the future payment conditional on no default, including the values of prepayment and mortgage continuation. LHS and RHS of the second Euler equation are the marginal cost and benefit of new house purchase, respectively. Today's marginal cost is the house price, while tomorrow's marginal benefit includes the implied rental rate $p_{ir} = u_h^B/u_c^B$ and the future net house value conditional on no default. The borrowing constraint decreases the marginal value of new mortgage if it is binding, but also lowers the marginal cost of new house purchase due to its value as collateral.

3.3 Financial Intermediary

Financial intermediaries channel the fund between the depositor and the borrower households. On one hand, an intermediary issues long-term loans to the borrower and hold the pool of mortgages as a risky asset. On the other hand, the debt on the balance sheet is the short-term risk-free bond issued to the depositor.

A key assumption here is that the depositor household cannot directly write a mortgage contract with the borrower household.²¹ Depositor's participation in risky asset investment is by holding the book equity in the intermediaries.²² The intermediaries facilitate risk sharing and maturity transformation that turns illiquid mortgage into liquid bond. However, there is a moral hazard problem that limits the mortgage credit supply. The agency problem will impose a constraint on the intermediary's problem and motivate an endogenous leverage.

Concretely, there is a unit mass of intermediaries owned by the depositor household. On the balance sheet of intermediary i at the end of period t , the holding of mortgage backed security a_{it}^I and the risk-free deposit b_t^I are the asset and the liability, respectively. The end-of-period balance sheet identity of intermediary i requires that the net worth n_{it}^I plus the liability equals to the asset.

$$n_{it}^I + b_{it}^I = a_{it}^I \quad (3.12)$$

At the beginning of period $t + 1$, an intermediary earns risky return $R_{m,t+1}$ on the asset and is obliged to pay the gross interest on the risk-free bond $R_{b,t}$ to the depositor. The law of motion of the net worth is

$$n_{i,t+1}^I = R_{m,t+1} a_{it}^I - R_{b,t} b_{it}^I \quad (3.13)$$

²⁰ Because ω^* is optimized by the borrower household, the marginal effects of m_{-1}^B and h_{-1}^B through ω^* are second-order and vanishing in Euler equations.

²¹ Mortgage origination usually requires expertise in credit evaluation and document verification that an individual depositor is not equipped with (Levitin and Wachter, 2011). This fact motivates the assumption of depositor's limited participation in the risky asset market, so intermediation is essential to risk sharing between the borrower and depositor household.

²² The model doesn't characterize the securitization market explicitly. The transfer between intermediary and the depositor household is lump-sum. Share of loans that were not held by depository institutions from 2001 to 2007 (or the share of loans that were securitized) slightly increased from 61% to 66%. Hence, it is not the growth of securitization, but the shift of mortgage composition that contributes to the cumulating mortgage risk.

Combine (3.12) and (3.13) to express the law of motion in terms of n_{it}^I and a_{it}^I .

$$n_{i,t+1}^I = (R_{m,t+1} - R_{b,t+1})a_{it}^I + R_{b,t}n_{it}^I \quad (3.14)$$

I assume that an intermediary faces binary turnover risk at the beginning of each period. The shock is *iid* across intermediaries and time. Dividend is not paid to the depositor household and the intermediary keeps accumulating net worth, until the intermediation business is terminated and the terminal net worth is brought back to the depositor household. The turnover probability is $1 - \psi$. By the Law of Large Number, it is also the fraction of intermediaries going back to the household as a lump-sum transfer. The depositor household will replace an exiting intermediary with a new one, and endow it with the start-up net worth specified below. The value of an intermediary i is the expected discounted terminal net worth

$$V_{it}^I = E_t \sum_{j=0} \Lambda_{D,t,t+1+j} (1 - \psi) \psi^j n_{i,t+1+j}^I \quad (3.15)$$

where $\Lambda_{D,t,t+1+i}$ is the depositor's stochastic discount factor specified later. The intermediary is subject to the incentive constraint due to the moral hazard problem. At the end of each period, the intermediary can divert and liquidate a fraction θ^I of the asset a_{it}^I , terminate the intermediation business and bring the liquidation value to the household. The incentive constraint that prevents an intermediary from asset diversion in equilibrium.

$$V_{it}^I \geq \theta^I a_{it}^I \quad (3.16)$$

θ^I controls the severity of the incentive problem. The higher θ^I is, the easier to divert asset and the tighter the constraint is. I interpret θ^I as the lending condition. Given the law of motion of the aggregate states, an intermediary maximizes the net worth at exit subject to the incentive constraint (3.16), and the law of motion (3.14). The recursive formulation of intermediary i 's problem is as follows.²³

$$\begin{aligned} V^I(a_i^I, n_i^I; S) &= \max_{a_i^{I'}} E \Lambda_D' [(1 - \psi)n_i^{I'} + \psi V^I(a_i^{I'}, n_i^{I'}; S')] \\ \text{s.t. (IC): } &V^I(a_i^{I'}, n_i^{I'}; S') \geq \theta^I a_i^{I'} \\ (LOM_n): &n_i^{I'} = (R_m' - R_b')a_i^I + R_b n_i^I \\ (LOM_S): &S' = \mathbf{K}(S) \end{aligned} \quad (3.17)$$

where $\Lambda_D' = \beta_D u_c^{D'} / u_c^D$ is the stochastic discount factor of the depositor household. Since the problem is linear in the state variables, I guess and verify the value function is linear in the states

$$V^I(a_i^I, n_i^I; S) = \nu_m(S)a_i^I + \nu_n(S)n_i^I \quad (3.18)$$

where $\nu_m(S)$ and $\nu_n(S)$ are time-varying due to their dependence on the aggregate states. ν_m is the expected marginal value of asset a_{it} , while ν_n is the expected marginal value of net worth. Because the incentive constraint is imposed for all future states, let the Lagrange multiplier be $\Lambda_D' \psi \lambda^I$. The optimality condition with respect to $a^{I'}$ implies

$$\nu_m' = \frac{\lambda^I}{1 + \lambda^I} \theta^I \quad (3.19)$$

²³ Without loss of generality, I use the formulation with asset and net worth as the sufficient statistics of an intermediary's balance sheet.

ν'_m is a transformation of the multiplier that controls whether (IC) is binding or slack. Substitute ν'_m for λ^I , the complementary slackness condition can be written as

$$\nu'_m(\phi' n_i^{I'} - a_i^{I'}) = 0, \quad \phi' n_i^{I'} - a_i^{I'} \geq 0, \quad \nu'_m \geq 0, \quad \text{where } \phi' \equiv \frac{\nu'_n}{\theta^I - \nu'_m} \quad (3.20)$$

ϕ' is a maximum endogenous leverage implied by the incentive constraint. It is interpreted as the marginal propensity to hold asset $a_i^{I'}$ with respect to the future net worth $n_i^{I'}$, when the constraint binds.²⁴ To verify that the value function has the conjectured form, replace the future value by (3.18) and substitute $\nu'_m a_i^{I'}$ by (3.20) in the value function (3.17). The undetermined coefficients are matched and need to solve the following equations recursively.

$$\begin{aligned} \nu_m &= E\Lambda'_D \Omega' (R'_m - R_b) \\ \nu_n &= E\Lambda'_D \Omega' R_b \\ \text{where } \Omega' &= 1 - \psi + \psi(\phi' v'_m + \nu'_n) \end{aligned} \quad (3.21)$$

$\Lambda'_D \Omega'$ is intermediary's pricing kernel that takes into account the consumption growth, the leverage and the turnover risk. The first equation equates the marginal value of asset and the expected excess return at optimum. Whenever $\nu_m > 0$, there is discrepancy between the ability and the willingness to lend. ν_m thus measures the inability to arbitrage away the excess return, or liquidity premium. If the incentive constraint never binds in the future $\nu'_m = 0$ and $\nu'_n = 1$ for all future states. The second condition implies unit value of the net worth $\nu_n = 1$.

The optimal conditions (3.21) are identical across intermediaries. Since the analysis examines the implication of the moral hazard problem, I focus on the case where, under relevant parametric values, the incentive constraint always binds around a local region of the steady state: $a_{it}^I = \phi_t n_{it}^I$. As the leverage ratio in (3.20) is identical across intermediaries, aggregation yields $a_t^I = \phi_t n_t^I$, with $a_t^I = \sum_i a_{it}^I$ and $n_t^I = \sum_i n_{it}^I$.

The aggregate law of motion of the net worth include two parts, the net worth of the remaining intermediaries and the start-up net worth of the new intermediaries. Aggregating (3.14) yields the net worth held by the existing intermediaries. I assume the start-up net worth is a fraction $\kappa/(1 - \psi)$ of the last period's mortgage at current value $q_t m_{t-1}^B$.²⁵ The law of motion of aggregate net worth is the weighted average net worth of the remaining and the new intermediaries.

$$n_t^I = \psi[(R_{mt} - R_{b,t-1})a_{t-1}^I + R_{b,t-1}n_{t-1}^I] + \kappa q_t m_{t-1}^B \quad (3.22)$$

3.4 Depositor Household

The depositor household is more patient than the borrower household, so it is the credit supplier through the intermediary in equilibrium. The depositor can save by means of the risk-free bond issued by financial intermediaries. I assume the depositor household consumes housing service flow from the exogenous house endowment $\bar{h}^D$, and it pays for the house maintenance each period. The exogenous house demand can be interpreted as a result of segmentation of borrowers' and depositors' house market. The depositor has the same class of preference over durable and non-durable consumption as the borrower household, $u(c^D, \bar{h}^D)$.

²⁴ As the multiplier is non-negative, a binding incentive constraint requires $\nu'_m \in (0, \theta^I)$. The lower bound comes from (3.19), while the upper bound is derived from $\phi' > 0$.

²⁵ Defining the start-up transfer as a share of last period's asset doesn't qualitatively change the law of motion of the aggregate net worth, while the current setup doesn't require to keep track of the asset and shrinks the state space by one dimension.

The assumption on the utility form is to minimize the impact from heterogeneity in preferences and to emphasize the role of the friction in the intermediation.

Upon receiving the labor income y_t^D and payoff from the risk-free bond b_{t-1}^D , the depositor's fund covers the non-durable consumption c_t^D , the expenditure on house maintenance $(1-\bar{\omega})p_t\bar{h}^D$, lump-sum tax T_t^B and the new purchase of the risk-free bond b_t^D . Same as the borrowers, the agents in the depositor household face idiosyncratic house price risk. Because the depositors have no mortgage debt and don't make default decision, the maintenance cost is deterministic from the perspective of the household.

Let Γ_t denote the net lump-sum transfer from the intermediaries, including the outflow of the start-up net worth to fund new intermediaries and the inflow of the net worth brought back by exiting intermediaries. The flow-of-fund constraint at the end of period t is

$$\begin{aligned} c_t^D + (1-\bar{\omega})p_t\bar{h}^D + b_t^D &\leq y_t^D - T_t^D + R_{b,t-1}b_{t-1}^D + \Gamma_t \\ \text{where } \Gamma_t &= (1-\psi)[(R_{mt} - R_{b,t-1})a_{t-1}^I + R_{b,t-1}n_{t-1}^I] - \kappa q_t m_{t-1}^I \end{aligned} \quad (3.23)$$

The end-of-period consolidated balance sheet of the depositor household is

$$c_t^D + (1-\bar{\omega})p_t\bar{h}^D + a_t^I \leq y_t^D - T_t^D + R_{mt}a_{t-1}^I \quad (3.24)$$

The internal risk-free debt as well as the transfer cancels out. Given the law of motion of the aggregate states, the depositor household maximizes her utility subject to the flow-of-fund constraint (3.23). The recursive formulation of the depositor household problem is

$$\begin{aligned} V^D(b_{-1}^D; S) &= \max_{c^D, \bar{h}^D} u(c^D, \bar{h}^D) + \beta_D EV^D(b^D; S') \\ \text{s.t. (FOF)} : c^D + (1-\bar{\omega})p\bar{h}^D + b^D &\leq y^D - T^D + R_{b,-1}b_{-1}^D + \Gamma \\ \text{(LOM}_S\text{)} : S' &= \mathbf{K}(S) \end{aligned} \quad (3.25)$$

which implies the optimality condition.

$$[b^D] : 1 = E\Lambda'_D R_b \quad (3.26)$$

where $\Lambda'_D = \beta_D u_c^{D'} / u_c^D$ is the stochastic discount factor of the depositor household.

3.5 Government

The government can collect lump-sum tax $T_t = T_t^B + T_t^D$ from the households to finance its expenditure. Assume the share of lump-sum tax paid by the borrower household is equal to the borrower's share of labor income, $\frac{T_t^B}{T_t} = \frac{y_t^B}{y_t}$. That is, the labor tax rate is constant across agents. On the expenditure side, the government spends an exogenous share g of the aggregate income, which is a deadweight loss in the model. The government can intermediate a fraction $\zeta_t \in [0, 1]$ of the total market mortgage outstanding $q_t m_t^B$. The government budget constraint is

$$\begin{aligned} g y_t + a_t^G &\leq T_t + R_{mt} a_{t-1}^G \\ \text{where } a_t^G &= q_t m_t^G = \zeta_t q_t m_t^B \end{aligned} \quad (3.27)$$

As is mentioned in [Bernanke \(2012\)](#), the Federal Reserve is limited by law to participate in large scale asset purchase other than the treasury and the federal agency securities. The main reason is to guarantee the functioning of the asset markets and to limit the expansion of the Fed's balance sheet. As a result, the path $\{\zeta_t\}_t$ is predetermined, rather than a choice variable by the government. It follows that I don't have to keep track of m_{t-1}^G and thus save one state variable.²⁶ In the benchmark calibration, the role of the government in the mortgage market will be turned off, $\zeta_t \equiv 0$.

3.6 Aggregate States and Law of Motion

The aggregate states of the economy consist of two parts, exogenous and endogenous states. The exogenous states include a vector of the aggregate income, the unitized risk and the lending condition, $S^{exo} = (y, \sigma_\omega, \theta^I)$. Denote the law of motion of the exogenous states as $S^{exo'} = K^E(S^{exo})$.

The unitized risk σ_ω , also known as the coefficient of variation, is defined in the model as the standardized dispersion of the idiosyncratic house price ω . Because a share of mortgage is tied to an equal share of house in the borrower household, an increase in the mortgage risk is mapped to an increase in the unitized risk of house price. I assume the idiosyncratic house price is Pareto distributed, $\omega \sim \text{Pareto}(\underline{\omega}, \gamma)$. The unitized risk σ_ω is parameterized by the shape γ of ω -distribution.

$$\sigma_\omega = \frac{\sqrt{\text{var}(\omega)}}{E(\omega)} = \frac{1}{\sqrt{\gamma(\gamma-2)}}, \quad \gamma > 2 \quad (3.28)$$

A smaller γ or a higher σ_ω corresponds to an increase in mortgage risk during the crisis.

The lending condition which is captured by the intermediary's share of divertible asset θ^I in the model affects the maximum credit supply. The smaller θ^I is, the larger the leverage ϕ and the maximum credit supply.

As to the endogenous states, the smallest set necessary to characterize the equilibrium keeps track of two endogenous variables. Denote the endogenous state variables $S^{endo} = (m_{-1}, \hat{b}_{-1})$ where m_{-1} and $\hat{b}_{-1}$ are the begin-of-period mortgage coupon and deposit repayment from the financial intermediary, respectively. Tracking $\hat{b}_{-1} \equiv R_{b,-1}b_{-1}$ instead results in a smaller state space than other alternatives. With the aggregate state $S = (S^{exo}, S^{endo})$, the controls $\mathcal{C}(S) \equiv [c^B(S), c^D(S), \nu_m(S), \nu_n(S), \lambda(S), p(S), q(S), R_b(S)]$ solve the functional equations from the optimality conditions, which I summarize in the Appendix B. The aggregate law of motion is

$$\begin{aligned} \hat{b}(S) &= R_b(S)[q(S)m(S) - n(S)] \\ \text{where } n(S) &= \psi[(Z(S) + \rho\pi(\omega^*(S))\mu q(S))m_{-1} - \hat{b}_{-1}] + \kappa q(S)m_{-1} \\ m(S) &= \frac{1}{q(S)} \{c^B(S) + [Z(S) + \rho\pi(\omega^*(S))\mu q(S)]m_{-1} \\ &\quad - [(1-\eta)\bar{\omega} + \eta z_1(\omega^*(S)) - 1]p(S)\bar{h}^B - y^B\} \\ S^{exo'} &= K^E(S^{exo}) \end{aligned} \quad (3.29)$$

The assumption on fixed house supply as a divisible Lucas tree implies that house prices capture all response to the change of market condition.²⁷ Since the depositor household consumes housing service only

²⁶ The government cannot issue public debt to finance the government expenditure in the model, but the risk-free debt issued by the intermediaries is available to the market, a perfect substitute of the government debt.

²⁷ As residential investment in the economy is not explicitly modeled, the effect captured by the house price can be considered an upper bound of the true impact in the context of inelastic supply side.

from the exogenous endowment, the borrower household is the marginal agent that determines the house price and h_t^B is interpreted as borrower's house asset holding share in the economy.

3.7 Equilibrium Concept

Denote the (shadow) price vector $\mathbf{p}(S)$, the collection of value functions $\mathbf{V}(S)$, the collection of policy functions $\mathbf{g}^i(S)$ for $i \in \{B, I, D\}$ as follows.

$$\begin{aligned}\mathbf{p}(S) &= \{q(S), p(S), R_b(S), \nu_m(S), \nu_n(S), \lambda(S)\} \\ \mathbf{V}(S) &= \{V^B(S), V^I(S), V^D(S)\} \\ \mathbf{g}^B(S) &= \{c^B(S), h^B(S), m^B(S), \omega^*(S)\} \\ \mathbf{g}^I(S) &= \{a^I(S), b^I(S)\}, \quad \mathbf{g}^D(S) = \{c^D(S), b^D(S)\}\end{aligned}$$

$\lambda(S)$ here denote the multiplier on the borrowing constraint to simplify notation. The recursive competitive equilibrium consists of the price vector $\mathbf{p}(S)$, the value functions $\mathbf{V}(S)$, the policy functions $\{\mathbf{g}^i(S)\}_{i \in \{B, I, D\}}$, and the aggregate law of motion $\mathbf{K}(S)$, such that

- Borrower: given $\mathbf{p}(S)$ and $\mathbf{K}(S)$, $V^B(S)$ and $\mathbf{g}^B(S)$ solve the problem (3.10).
- Intermediary: given $\mathbf{p}(S)$ and $\mathbf{K}(S)$, $V^I(S)$ and $\mathbf{g}^I(S)$ solve the problem (3.17).
- Depositor: given $\mathbf{p}(S)$ and $\mathbf{K}(S)$, $V^D(S)$ and $\mathbf{g}^D(S)$ solve the problem (3.25).
- The price vector $\mathbf{p}(S)$ clears the markets:
 - (a) House market: $h^B(S) = 1 - \bar{h}^D = 1$
 - (b) Bond market: $b^I(S) = b^D(S)$.
 - (c) Credit market: $(1 - \zeta)q(S)m^B(S) = a^I(S)$.
 - (d) Consumption good market:

$$c^B(S) + c^D(S) + (1 - \bar{\omega})p(S) + \eta[\bar{\omega} - z_1(\omega^*(S))]p(S)h^B(S) = (1 - g)y$$

- The perceived law of motion $K(S)$ is consistent with the actual law of motion (3.29).

4 General Equilibrium Implication of Credit Limits

The borrowing and the lending limits are both endogenous in the model, while the equilibrium mortgage credit and the mortgage spread are determined by the interaction of the credit limits. When the borrowing (lending) constraint is more binding, the equilibrium mortgage credit attaches more weight to the credit demand (supply), due to the widening discrepancy between the ability and the willingness to borrow (lend).

The objective is to show the determinants of mortgage credit and the mortgage spread. Particularly, I examine the role of the mortgage default option. I consider two cases of ω -distribution where the default decision is either time-invariant (exogenous default) or state-dependent (endogenous default). I will focus on the steady state analysis to characterize the interaction of the credit limits in the starkest way.²⁸

²⁸ The component of the mortgage spread associated with risk and uncertainty will vanish at the steady state, but the component associated with funding liquidity will survive. The steady state analysis can be interpreted as examining the long-run or permanent effects. The effects will survive, when we go to the fully dynamic and stochastic model.

To illustrate the mechanism, I make the following assumptions for simplification: (1) Agents have perfect foresight. (2) $u^i(c, h) = c + \alpha \log(h)$ for $i \in \{B, D\}$. The stochastic discount factor is independent of consumption growth. (3) $h^B = h^D = 1$. The amount of segmented houses is rescaled for illustrative purpose. (4) $\psi = 0$. An intermediary operates only for one period without consideration of continuation. (5) $\eta = 1$. The foreclosure technology cannot recover any house value in case of default. (6) $\mu = 0$ and $\rho = 1$. The mortgage contract is short-term, so prepayment is trivial in the context. The borrowing constraint becomes $qm^B \leq \theta^B ph^B$. (7) $g = 0$. There is no government expenditure.

4.1 Determinants of Mortgage Supply and Demand

The optimality condition of the depositor (3.26) yields the risk-free rate $R_b = \beta_D^{-1}$. Combined with the optimality of the intermediary, I solve for $\nu_n = 1$ and $\nu_m = \beta_D R_m - 1$. When $\nu_m = 0$, the incentive constraint is slack and the return spread $R_m - R_b$ is zero. The maximum leverage (3.20) at the steady state is reduced to

$$\hat{\phi}(R_m) = \frac{1}{\theta^I - \nu_m}, \text{ where } \nu_m = \beta_D R_m - 1 \geq 0 \quad (4.1)$$

Denote the wealth before transfer to new intermediaries and the start-up net worth to be $W^D = y^D + R_m a^I$ and n_{new}^I , respectively. I assume the transfer rule to support new intermediaries is a fraction $\tau \in (0, 1)$ of the before-transfer wealth, $n_{new}^I = \tau W^D$. With zero probability of continuation, the net worth all comes from new intermediaries, $n^I = n_{new}^I$. The maximum mortgage credit supply is pinned down by the incentive constraint $a^I \leq \phi n^I$, which is an increasing function of R_m .

$$a^I \leq \frac{\hat{\phi}(R_m) \tau}{1 - \hat{\phi}(R_m) \tau R_m} y^D \equiv \bar{S}(R_m) \quad (4.2)$$

The inequality is strict, when the mortgage credit is determined by the demand side.

For the case of exogenous default, assume ω is degenerate to a Bernoulli distribution. Let $z_0 = \pi$ and $z_1 = 1 - \delta$ to parameterize the distribution. For the case of endogenous default, I maintain the assumption that ω is Pareto distributed.

Case 1: ω is Bernoulli. By the borrower's optimality condition (3.11), if default decision is independent of the state of the economy, the house price is reduced to

$$p = \frac{\alpha}{1 - \beta_B(1 - \delta) - \theta^B \lambda_l^B}, \text{ where } \lambda_l^B = 1 - \beta_B R_m \geq 0 \quad (4.3)$$

The house price is the present discounted rental value α , with the discount factor adjusted for the collateral value of house and maintenance. The maximum credit demand is determined by the borrowing constraint, which is decreasing in R_m if the constraint binds.

$$qm^B \leq \frac{\alpha \theta^B}{1 - \theta^B(1 - \beta_B R_m) - \beta_B(1 - \delta)} \equiv \bar{D}(R_m) \quad (4.4)$$

The inequality is strict, when the mortgage credit is determined by the supply side.

Case 2: ω is Pareto. When the default decision is state-dependent, the assumption that ω is Pareto distributed, $\omega \sim \text{Pareto}(\underline{\omega}, \gamma)$, will simplify the illustration. The cumulative distribution function is $F_\omega(\omega) =$

$1 - (\underline{\omega}/\omega)^\gamma$ for $\omega \geq \underline{\omega}$. I assume parameter γ is large enough $\gamma > \gamma_0 > 1$, implying that sufficient mass of borrowers are subject to positive maintenance cost by drawing ω in the unit interval.²⁹ It is convenient to note that $z_0 = \pi$ and z_1 have the following relationship.³⁰

$$\beta_B z_1(\omega') = \frac{\gamma}{\gamma-1} \cdot \beta_B \pi(\omega') \cdot \omega' = \frac{\gamma}{\gamma-1} \cdot (1 - \lambda_l^B) q \cdot \frac{m^B}{p'} \quad (4.5)$$

where the first equality is based on the property of Pareto distribution, and the second equality uses the default threshold (3.5) and the optimality condition $[m^B]$ in (3.11). I solve for the maximum demand of mortgage credit $\theta^B p$ using (3.4), $[h^B]$ in (3.11) and (4.5).³¹

$$\begin{aligned} qm^B &\leq \frac{\alpha \theta^B}{1 - \underbrace{\theta^B \lambda_l^B}_{\text{Housing Collateral Channel}} - \underbrace{\theta^B \frac{\gamma}{\gamma-1} (1 - \lambda_l^B)}_{\text{Mortgage Default Channel}}} \\ &= \frac{\alpha \theta^B}{1 - \frac{\theta^B}{\gamma-1} (\gamma - \lambda_l^B)} \equiv \bar{D}^d(R_m), \text{ where } \lambda_l^B = 1 - \beta_B R_m \end{aligned} \quad (4.6)$$

The inequality is strict, when the mortgage credit is determined by the supply side.

Different from the case of exogenous default, $\bar{D}^d(R_m)$ is increasing in R_m , but at a slower rate than $\bar{S}(R_m)$ if $\gamma > \gamma_0$. The maximum mortgage credit demand in (4.6) is dependent on the borrowing constraint, due to the following two channels that relate the house price p and the shadow cost of borrowing λ_l^B .

- *Housing Collateral Channel.* Houses are used as collateral in mortgage borrowing, leading to a lower marginal cost of house purchase. A tighter borrowing constraint (higher λ_l^B) increases the collateral value $\lambda_l^B \theta^B p$, thus increasing the house price p .
- *Mortgage Default Channel.* A tighter borrowing constraint implies a smaller marginal benefit of borrowing $(1 - \lambda_l^B) q$. As the benefit and the cost of mortgage borrowing are equated at the margin, the marginal cost of borrowing $\beta_B \pi'$ will be smaller, implying the probability of default is expected higher. The continuation value of house z_1' is dampened by a higher default threshold, thus decreasing the house price p .

The channels summarize two distinct roles of the borrowing constraint in the collateralized borrowing with a default option. The housing collateral channel shows the impact of collateral value equally exerted on both current and future house price, and is already presented in the exogenous default case in (4.4). The mortgage default channel emphasizes how expecting higher probability of future default will decrease the current house price by dampening its continuation value. In the simplified model, (4.6) implies that the force through the mortgage default channel is stronger than the force through the housing collateral channel, leading to an upward sloping curve.

²⁹ The assumption is a technical assumption to guarantee, at least, the existence of the expectation, and possibly the existence of higher moments. If $\gamma > n$, then the n -th raw moments exist. Concretely, $\gamma > \gamma_0$ implies that the tail distribution is not too fat. This is important to derive the result below. Given ω_0 , a bigger γ lowers the expected value $\frac{\gamma}{\gamma-1} \omega_0$, leading to higher probability of paying a positive maintenance cost. I need the condition of sufficient mass to constrain the expected value of house z_1 from above, so that the house stochastically depreciates and the maintenance cost is paid on aggregate by the borrower household.

³⁰ Note $z_0(\omega) = \pi(\omega) = \frac{\omega^\gamma}{\omega^\gamma}$ and $z_1(\omega) = \frac{\gamma}{\gamma-1} \frac{\omega^\gamma}{\omega^{\gamma-1}}$.

³¹ For the case where the borrowing constraint binds ($\lambda_l^B > 0$), use $qm^B = \theta^B p$ and solve for p from the optimality condition $[h^B]$. $\bar{D}^d(R_m)$ is then $\theta^B p$. For the case where the borrowing constraint slacks ($\lambda_l^B = 0$), directly solve for qm^B from $qm^B \leq \theta^B p = \theta^B (\alpha + \frac{\gamma}{\gamma-1} qm^B)$ and define the transformed RHS as $\bar{D}^d(R_m)$. The form of the maximum demand of mortgage credit $\bar{D}^d(R_m)$ is the same.

4.2 Steady State with Exogenous Default

At the steady state equilibrium, the credit market clears, $a^I = qm^B$. There are three regimes to consider, depending on whether the incentive constraint, the borrowing constraint, or both constraints will bind at the steady state. Because the Lagrange multipliers are non-negative, $R_m \in [\beta_D^{-1}, \beta_B^{-1}]$ is the domain of the asset return, beyond which there is either zero supply or demand of mortgage credit. The equilibrium is summarized in the following proposition.

Proposition 4.1. *The threshold function*

$$\Theta(R_m) = \frac{1 - \beta_B(1 - \delta)}{\frac{\alpha}{\hat{\phi}(R_m)\tau y^D} [1 - \hat{\phi}(R_m)\tau R_m] + (1 - \beta_B R_m)} \quad (4.7)$$

is increasing in R_m . At the steady state,

- When $\theta^B < \Theta(\beta_D^{-1})$, $\bar{S}(R_m) > \bar{D}(R_m)$ for all $R_m \in [\beta_D^{-1}, \beta_B^{-1}]$. The intermediary is the unconstrained agent that prices the risky asset, $R_m = \beta_D^{-1}$.
- When $\theta^B > \Theta(\beta_B^{-1})$, $\bar{S}(R_m) < \bar{D}(R_m)$ for all $R_m \in [\beta_D^{-1}, \beta_B^{-1}]$. The borrower is the unconstrained agent that prices the risky asset, $R_m = \beta_B^{-1}$.
- When $\Theta(\beta_D^{-1}) \leq \theta^B \leq \Theta(\beta_B^{-1})$, $\exists! R_m \in [\beta_D^{-1}, \beta_B^{-1}]$ that solves $\bar{S}(R_m) = \bar{D}(R_m)$.

Proof. See Appendix. □

In the first (second) case, only the credit demand (supply) constraint is relevant, and the mortgage rate is characterized by the inverse of the unconstrained agent's discount factor. A local change of the tightness of the credit supply (demand) by θ^I (by θ^B) won't affect the equilibrium prices and quantities.

The interesting case is the last one in which both the supply and demand constraints bind. This is not a knife-edge case, because both the leverage that limits the credit supply (4.2) and the housing price that limits the credit demand (4.4) are determined endogenously. With quasi-linear preferences, the mortgage demand (supply) is perfectly elastic when the borrowing (incentive) constraint is slack, and is perfectly inelastic when it binds. The equilibrium in the case of two binding constraints looks *as if* the asset return is pinned down by an upward sloping supply curve $\bar{S}(R_m)$ and a downward sloping demand curve $\bar{D}(R_m)$ (see Figure 9a). Combine (4.1) and (4.3).

$$\beta_D^{-1} \nu_m + \beta_B^{-1} \lambda_l^B = \beta_B^{-1} - \beta_D^{-1} \quad (4.8)$$

The negative relationship of the shadow costs that measure the supply and demand funding illiquidity summarizes the general equilibrium linkage of credit limits. Suppose there is an exogenous increase in θ^B that shifts the credit demand curve rightward and increases the maximum credit demand. The decrease in the shadow cost of borrowing is associated with an increase in the shadow value of lending, leading to the increase in the maximum credit supply in equilibrium. On the other hand, an exogenous change of θ^I will trigger similar general equilibrium adjustment of credit limits.

The equation $\theta^B = \Theta(R_m)$ summarizes how an exogenous shift of the incentive (borrowing) constraint by θ^I (by θ^B) affects the asset return and thus the mortgage spread. The impact of a 5 percent shift of credit constraints is illustrated in Figure 9a. Lending relaxation (smaller θ^I) increases the intermediary's leverage, thus slackening the mortgage credit supply. Borrowing relaxation (bigger θ^B) increases the collateral value of house, which relaxes the mortgage credit demand. We come to the following corollary on the spread.

Corollary 4.2. *For $\theta^B \in (\Theta(\beta_D^{-1}), \Theta(\beta_B^{-1}))$, The premium $R_m - R_b$ increases in θ^B and θ^I .*

4.3 Steady State with Endogenous Default

Similar to Proposition 4.1, Proposition 4.3 shows the determinants of the asset return and thus the mortgage spread, but in the context of endogenous default.

Proposition 4.3. *If $\gamma > \gamma_0 \equiv \frac{\beta_B \tau y^D}{(\beta_D + \tau)\alpha} + 1$, then the threshold function*

$$\Theta^d(R_m) = \left[\frac{\alpha}{\hat{\phi}(R_m) \tau y^D} [1 - \hat{\phi}(R_m) \tau R_m] + \frac{1}{\gamma - 1} [\gamma - (1 - \beta_B R_m)] \right]^{-1} \quad (4.9)$$

is increasing in R_m . At the steady state,

- *When $\theta^B < \Theta^d(\beta_D^{-1})$, $\bar{S}(R_m) > \bar{D}^d(R_m)$ for all $R_m \in [\beta_D^{-1}, \beta_B^{-1}]$. The intermediary is the unconstrained agent that prices the risky asset, $R_m = \beta_D^{-1}$.*
- *When $\theta^B > \Theta^d(\beta_B^{-1})$, $\bar{S}(R_m) < \bar{D}^d(R_m)$ for all $R_m \in [\beta_D^{-1}, \beta_B^{-1}]$. The borrower is the unconstrained agent that prices the risky asset, $R_m = \beta_B^{-1}$.*
- *When $\Theta^d(\beta_D^{-1}) \leq \theta^B \leq \Theta^d(\beta_B^{-1})$, $\exists! R_m \in [\beta_D^{-1}, \beta_B^{-1}]$ that solves $\bar{S}(R_m) = \bar{D}^d(R_m)$.*

Proof. See Appendix. □

The impact of a 5 percent exogenous shift of the credit constraints by θ^I or θ^B is illustrated in Figure 9b. Borrowing relaxation with endogenous default increases the mortgage credit and the mortgage spread, while lending relaxation decreases the mortgage credit and the mortgage spread, a result different from the case of exogenous default. Note that the state-dependent default option flattens the curve of maximum credit demand. Lending relaxation by a 5 percent decrease in θ^I results in a bigger response of the mortgage spread in the case of endogenous default than exogenous default. Moreover, the response of the borrowing constraint to a 5 percent increase in θ^B is much larger with endogenous default than exogenous default, attributed to the dominant impact of mortgage default channel. Nevertheless, both cases yield the same comparative statics of the mortgage spread.

Corollary 4.4. *For $\theta^B \in (\Theta^d(\beta_D^{-1}), \Theta^d(\beta_B^{-1}))$, The premium $R_m - R_b$ increases in θ^B and θ^I .*

The steady state analysis shows that the maximum credit supply and the maximum credit demand are interdependent, but is silent on the model dynamics and the role of risk and uncertainty. The next section will examine the role of the current and the future constraints.

5 Calibration

The model is calibrated at annual frequency. The structure of the model allows me to solve the steady state values of endogenous variables quasi-analytically. Under the assumption that ω is Pareto distributed, I prove the steady state is unique. The equilibrium conditions and the steady state are reported in Appendix B. I summarize the parameters for calibration in Table 3. This section focuses on the sources and the methods to determine the parameters in calibration.

5.1 Exogenous Processes

The exogenous aggregate shock process consists of three independent components, the unitized risk σ_ω , the lending condition θ^I and the income y .

5.1.1 Unitized risk

I use foreclosure or delinquency rates to discipline the process of unitized risk σ_ω .³² Assume $\log \gamma_t$ follows an AR(1) process.

$$\log \gamma_t = (1 - \rho_{lg}) \log \gamma_N + \rho_{lg} \log \gamma_{t-1} + \sigma_{lg} \epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (5.1)$$

Time-varying unitized risk is thus captured by the shift of the Pareto shape γ . To reduce the number of calibrated parameters and to make the households as symmetric as possible, I assume $E(1 - \omega)$ is time-invariant and match the annual depreciate rate of 2% (Tuzel, 2010). This allows me to pin down the scale parameter ω , given each value of γ . Therefore, three parameters remains to calibrate: ρ_{lg} , γ_0 and σ_{lg} .

ρ_{lg} controls the persistence of the AR(1) process (5.1). In case of no serial correlation, the pre-crisis impact of increasing mortgage risk will be short-lived in housing bust. Considering the lengthening effect of foreclosure process (Cordell *et al.*, 2015), I conservatively set $\rho_{lg} = 0.6$ to match the enduring impact of default and map it into 2 years for the default rate to decrease by half from the spike in the boom-bust simulation, consistent with the average length of crisis in Elenev *et al.* (2016).

γ_N controls the default rate in the normal time and its logarithmic value is the unconditional mean of the AR(1) process. I calibrate γ_N to match a 1% default rate at the steady state, which results in the unitized risk in the normal time $\sigma_{\omega,L} = 0.285$ (equivalently $\gamma_N = 4.645$). The targeted mean default rate is a compromise of the delinquency and foreclosure rates from 2001 to 2006 (1.61% and 0.6% respectively). σ_{lg} controls the volatility of the default rate. I set $\sigma_{lg} = 0.030$ to relate the standard deviation of the default rate in the model to the geospatial standard deviation of the delinquency rate in Brown *et al.* (2012).³³

Because the borrower household can refinance mortgages each period at no cost, mortgage default becomes a cheap alternative to new borrowing to manage the mortgage stock. Without adjustment cost, the model turns out to overstate the default rate volatility in the bust episode and to outweigh the role of mortgage default. In quantitative analysis, I assume that the default threshold follows the *ad hoc* form to take into account a reduced-form adjustment cost of default.

$$\hat{\omega}_t^* = (1 - s_\omega) \omega_t^* + s_\omega \omega_{ss}^* \quad (5.2)$$

The fraction $s_\omega \in [0, 1]$ adopts the non-contingent threshold at the steady state ω_{ss}^* , while the remaining $1 - s_\omega$ makes default decision based on the optimal threshold ω_t^* in (3.5). The bigger s_ω is, the higher the adjustment cost of default and the less volatile the default rate in the bust episode.³⁴

I choose s_ω to match the spike of the default rate in the bust episode. I define the bottom 10% of the unconditional distribution $\log \gamma_t$ as the regime of mortgage crisis, with the crisis arrival rate borrowed from

³² Van Nieuwerburgh and Weill (2010) show the time series of unitized risk of house prices across metropolitan statistical areas (MSAs) from 1975 to 2007. They find the population-weighted unitized risk of house price across MSAs surged from 0.45 in 2000, to the historical height 0.57 in 2005, and slightly decreased to 0.53 in 2007. In this paper, unitized risk targets mortgage default. While calibrated unitized risks are smaller, the scale and trend are consistent with the literature (Van Nieuwerburgh and Weill, 2010; Landvoigt, 2016; Elenev *et al.*, 2016).

³³ Brown *et al.* (2012) estimate the geospatial standard deviation to be 0.5% for all loans and 1.1% for subprime loans, capturing the zipcode variation of delinquency rates within 91 MSAs. However, as is argued by the authors, the estimated values are regarded conservatively small, because each loan in the same zipcode is assigned an identical delinquency rate in the presence of house and mortgage heterogeneity. I assume and match a 5% standard deviation of the default rate at the census tract and block level. In the presence of 100 independent realizations (a reasonable ratio of the number of census tract and blocks to the number of zipcodes), it yields a 0.5% standard deviation of the default rate at the zipcode level.

³⁴ Consider a lottery with binary outcomes realized after the state-contingent default decision is made. With probability s_ω , an individual borrower faces infinite non-pecuniary utility cost of deviating from ω_{ss}^* , while with probability $1 - s_\omega$, a borrower incurs zero utility cost to adjust the threshold optimally. In the extreme case, $s_\omega = 0$ means the default threshold is optimal and fully flexible, while $s_\omega = 1$ indicates the threshold is fully rigid. The rigidity can be rationalized by adding a non-pecuniary utility cost of default to the benchmark model, with the details available in Appendix C.3.

Elenev *et al.* (2016). The 10th percentile of the log-shape distribution $\log \gamma_C$ is mapped to the pre-crisis peak of the unitized risk to match the spike of the default rate in boom-must simulation.

$$\log \gamma_C = \log \gamma_N + F_{N(0,1)}^{-1}(0.1)\sigma_{lg} \quad (5.3)$$

I calibrate a 9% default rate in the simulated crisis as a compromise of the delinquency rate and the foreclosure rate at the peak of the mortgage crisis (11.7% and 3.6% respectively). This yields $s_\omega = 0.95$ and the unitized risk in the crisis time $\sigma_{\omega,H} = 0.305$ (equivalently $\gamma_C = 4.430$), leading to a 6.8% increase in the unitized risk from the normal time. In Figure 10a, I illustrate two ω -distributions parameterized by $\sigma_{\omega,L}$ and $\sigma_{\omega,H}$, while in Figure 10b, I show how unitized risk increases from $\sigma_{\omega,L}$ to $\sigma_{\omega,H}$ in σ_ω -distribution.

5.1.2 Lending Condition

I use bank leverage to discipline the process of lending condition. Assume θ^I follows an AR(1) process.

$$\theta_t^I = (1 - \rho_{\theta^I})\theta_0^I + \rho_{\theta^I}\theta_{t-1}^I + \sigma_{\theta^I}\epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (5.4)$$

A higher ρ_{θ^I} indicates higher persistence of leverage. I use a panel of US banks from Compustat Capital IQ to match the persistence of leverage in logarithmic value. When the sample is restricted to the periods after formal enforcement of Basel I but before the financial crisis (1993-2007), the risk-weighted and unweighted leverages have similar levels of persistence (Table 4), irrelevant to the risk weighting methodologies. I set $\rho_{\theta^I} = 0.11$ to match the persistence of logarithmic leverage to 0.47 which is the persistence of tier 1 leverage in the data.

The unconditional mean θ_0^I controls the level of an intermediary's leverage. I jointly calibrate the mean divertible share $\theta_0^I = 0.481$ and the transfer share to the start-up intermediaries $\kappa = 0.0053$ to match two targets at the steady state: (1) the intermediary's leverage is 10; (2) the spread of risky over risk-free return is 255 basis points. The first target is roughly consistent with the median leverage of the banking sector from 1993 to 2007 in the data (Table 5).³⁵ The second target is mapped from the mean of Barclays MBS annual total return over the 1-year treasury rate from 1977 to 2017, and is quantitatively similar to the value in Landvoigt (2016). σ_{θ^I} controls the conditional volatility of the leverage. I set $\sigma_{\theta^I} = 0.037$ to match the standard deviation of simulated logarithmic leverage to 0.33, its counterpart of the risk-weighted bank leverage from 1993 to 2007 (Table 5).

To determine the size of the pre-crisis relaxation of lending condition, I map the lending condition at the end of housing boom $\theta_C^I < \theta_0^I$ to a 5% increase in the intermediary's leverage at the steady state, and consider a relaxing path of θ_t^I deviating from its mean in the simulated housing boom. The percentage change is borrowed from the median growth of risk-weighted leverage from 2004 to 2007 (Table 6), the period when mortgage credit experienced wild growth due to non-agency securitization (Tables 1 and 2). This yields the pre-crisis lending condition $\theta_C^I = 0.436$, which corresponds to a 9.4% decrease of θ^I .

5.1.3 Income

I assume the logarithmic nominal income $\log Y_t$ takes the form $\log Y_t = G_n(\log Y_{t-1}) + g_t$ where $G_n(\cdot)$ is an n -th order polynomials of the lag term and g_t is the stationary component of income growth rate. The

³⁵ The choice of the leverage ratio can be interpreted as a compromise of lower leverages among commercial banks and higher leverages among investment banks (Gertler *et al.*, 2016).

polynomial term is used to separate the stochastic trend of the process from the stationary process. I use a quadratic form $G_2(\cdot)$ and use the regression residuals to estimate the process of g_t which is assumed to follow an AR(1) process.

$$g_t = (1 - \rho_y)\bar{g}_y + \rho_y g_{t-1} + \sigma_y \epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (5.5)$$

where ρ_y is the persistence and σ_y is the conditional volatility of the income growth rate. I use the annual series of US GDP per capita from BEA's NIPA Table 7.1 to estimate the income growth process. The series dates back to 1929, but I exclude the period of the World War II (1940-1945). The estimation yields the persistence $\rho_y = 0.493$ and the standard deviation $\sigma_y = 3.3\%$. I exponentiate the process g_t to construct the stationary aggregate income $y_t = \exp(g_t)$. Because the model is scalable by the income level, without loss of generality, I make the normalization $\bar{g}_y = 0$, which results in the income at the steady state to be unity.

5.2 Financial Intermediary

The intermediary's continuation probability is set to $\psi = 0.75$ for an expected length of operation of 4 years, consistent with the assumption in [Queralto \(2013\)](#) and [Gertler and Kiyotaki \(2015\)](#). There are three parameters associated with the long-term defaultable mortgage contract. I set the mortgage decay rate to $\mu = 0.954$ to match the duration of a fixed-rate mortgage and the mortgage contract in the model. The share of principal component is set to $\varphi_0 = 0.63$.³⁶ It means that on average, 63% of the annual payment is regarded as the mortgage principal, and the pool of mortgages with various origination and contract length is approximated by a duration-matched geometric mortgage contract. The foreclosure loss rate $\eta = 0.32$ is based on the sales loss within one year from foreclosure estimated by [Campbell et al. \(2011\)](#).

The mortgage prepayment rate is derived from the loan-level data from the Columbia Collateral Files generated by Wells Fargo Bank. I calculate the single monthly mortality rate (SMM) and then convert it to the conditional prepayment rate (CPR).³⁷ The estimated prepayment rate $1 - \rho = 15\%$ is the time-series mean of CPR. As to the limit of credit demand, I set the maximum admissible loan-to-value ratio to $\theta^B = 0.80$. The ratio is consistent with the finding of mass bunching at the rule-of-thumb threshold of risky loans ([Greenwald, 2017](#)).³⁸

5.3 Household

I use the share of US real housing service and utilities expenditure in the real personal consumption expenditures from BEA to set $\alpha = 0.18$. I adopt the estimated value of the borrower's house share $h^B = 0.48$ and income share $\tau^B = 0.45$ from [Landvoigt \(2016\)](#) whose estimates are based on the Survey of Consumer Finances from 1983 to 2007. The split of borrowers and depositors is based on the rule whether the net fixed-income position of a household is positive. The government share of expenditure $g = 16.3\%$ matches

³⁶ I parameterize a fully amortized fixed-rate mortgage with a real annual interest rate $r_{FRM} = 3.3\%$ and a loan term $T_{FRM} = 30$. The real mortgage rate is the sum of the real risk-free rate 1.2% plus the mean of the empiric mortgage spread. The cash flows are discounted by the mortgage rate r_{FRM} . The decay rate μ yields a duration of 13.1 years of the geometric mortgage, consistent with its counterpart of the fixed-rate mortgage. The share of principal component is based on same fixed-rate mortgage.

$$\varphi_0 = \frac{Loan}{Payment \cdot T_{FRM}}, \text{ with } Payment = \frac{r_{FRM}}{1 - (1 + r_{FRM})^{-T_{FRM}}} Loan$$

³⁷ Single monthly mortality rate is calculated on a monthly basis and is defined as the prepaid principal as the share of the begin-of-period principal balance net of the scheduled principal paid. CPR annualizes the monthly prepayment measure.

³⁸ The reason why there is mass at 80% is associated with the fact that a borrower can avoid incurring additional expense of private mortgage insurance (PMI) by keeping the mortgage balance below 80%. Congress requires the Government-Sponsored Enterprises to obtain a private market credit enhancement by PMI for the high-leverage loans ([Bhutta and Keys, 2018](#)).

the average exogenous government spending to GDP in the US national account. The share is borrowed from [Elenev et al. \(2016\)](#) which uses BEA’s NIPA Table 3.1 in calculation.

The depositor’s discount factor β_D prices the risk-free deposit in the model. [Elenev et al. \(2016\)](#) find the mean of one-year real risk-free rate to be 1.2% based on the the nominal interest rate and the Survey of Professional Forecasters. I target the risk-free rate at the steady state in the model to match the historical average in the data, which implies $\beta_D = 0.988$. The borrower’s discount factor β_B controls the mortgage credit demand and ultimately the house price. [Greenwald \(2017\)](#) estimates the ratio of median house price to the median income of the borrower to be 2.22 from the 1998 Survey of Consumer Finance. I thus set $\beta_B = 0.904$ to match this empirical ratio to the model counterpart. Similar values are found in [Elenev et al. \(2016\)](#) and [Landvoigt \(2016\)](#). The risk aversion coefficient of the borrower household $\xi^B = 1$ follows the standard value in the literature, while the risk aversion coefficient of the depositor household $\xi^D = 20$ follows [Elenev et al. \(2016\)](#) to match the volatility of the mortgage-to-GDP ratio which is 4.2% from 1985 to 2014.

6 Quantitative Analysis

This section illustrates how the model transmits and amplifies the shocks of income, unitized risk or lending condition to mortgage debt, house prices, and economic activities. The quantitative results are obtained by conducting pruned third-order perturbation with binding credit constraints.³⁹ Simulation of the boom-bust episodes and generalized impulse responses to the model’s fundamental shocks use the stochastic instead of deterministic steady state as the reference point.⁴⁰

6.1 Model Assessment: Observed vs. Simulated Moments

To evaluate the performance of the model, I compare the observed moments generated by data series and the simulated moments by the calibrated model. I report three sets of endogenous variables associated price, quantity, and interest rate and spread. The report focuses on the relative price and quantity ratios whose economic interpretations are directly associated with the data counterparts. On the other hand, data series are inflation adjusted and detrended for stationarity purpose, before calculating the price or quantity ratios and estimating comparable observed moments. The sample before 2000 is used to estimate the observed moments to prevent the detrended series from overweighing the boom episode.

Table 7 report the observed and simulated means and standard deviations. Overall, the model performs well to generate plausible averages and variations in response to aggregate risks. House price and implied rental price relative to the income produce reasonable housing affordability measures (2.2 for price-to-income ratio and 0.16 for rent-to-income ratio), close to the observed ratios 2.1 and 0.14, respectively. The price-to-rent ratio indicates the model slightly overestimates the house price and underestimates the rental price. This yields a price-to-rent ratio of 13.9 compared with 15.7 in the data.

The calibrated mortgage payment-to-disposable income ratio is 0.069, slightly bigger than the observed ratio of total debt payment to disposable income 0.056. The new mortgage issuance relative to the mortgage stock is 24.6% with a standard deviation of 3%, close to the observed counterparts of 26.4% and 3.54% respectively. Because the moments of default rate and leverage are targeted to pin down the parameters of

³⁹ While first-order perturbation cannot account for risk premiums due to certainty equivalence, second and higher-order perturbation is necessary to generate non-zero and time-varying risk premiums.

⁴⁰ With time-varying unitized risks, ergodic distribution of endogenous variables may move away from the deterministic steady state ([Fernández-Villaverde et al., 2011](#)).

the unitized risk and the lending condition, the calibrated and observed moments are close by construction.

The model is successful to achieve high volatility of risky asset return R_m relative to the implied mortgage rate $R_q = q^{-1} + \mu$. R_m is predicted 10 times more volatile than R_q , compared to 5 times found in the data. The standard deviations of the excess return $R_m - R_{b,-1}$ and the implied mortgage spread $R_q - R_{b,ss}$ are 4.1% and 0.4%, with the observed counterparts 6% and 0.5% respectively.⁴¹ From borrower's and intermediary's optimality conditions, R_q and R_m are associated with borrower's and intermediary's pricing kernels respectively. On one hand, the depositor household are more risk averse than the borrower household. On the other hand, the agency friction augments the cyclical of the pricing kernel. Both factors contribute to higher volatilities of risky return and the excess return than the mortgage rate and the mortgage spread.

Table 8 reports the serial correlations up to 4th order of the same set of variables. Overall, the simulated series have plausible persistence patterns compared to the data series. The model is successful to capture higher persistence of the mortgage rate and the mortgage spread than the risky asset return and the mortgage spread over the risk-free rate. The mortgage rate and its spread have first order serial correlations of 0.70 and 0.70, whose data counterparts are 0.35 and 0.64 respectively. The risky return and the excess return have first order serial correlations of 0.21 and -0.18 in the model series, while the persistence in the data series is -0.01 and -0.11 respectively.

6.2 Quantitative Experiment: Boom-Bust Episode

To understand the implication of the model in the boom-bust episode, I conduct a quantitative experiment with a particular joint realization of the aggregate shocks to reproduce the boom-bust dynamics around the Great Recession. Figure 11 plots a particular realized path of the aggregate shocks including the income y , unitized risk σ_ω and the credit lending condition θ^I . The paths of unitized risk and lending condition are disciplined by data, with details discussed in Section 5. To understand the importance of each shock and the source of macro fluctuation, I consider three alternative models, with only one of the three shocks at work in each economy.

In the simulation, I engineer the initial 9 periods to be hit by the unexpected shocks, mimicking the periods in the boom and bust before and after the Great Recession from 2001 to 2009. Starting from the steady state, the first six periods are hit by a sequence of positive income shocks to characterize the boom periods, followed by negative income shocks as the bust periods. The simulated income path replicates the dynamics of the stationary component of the real US GDP per capita in the same horizon.⁴² To capture the growing mortgage risks associated with the exponential growth of non-agency securitization, the unitized risk increases from the steady state from period 4 at an increasing rate and reaches the peak at period 8. To take the relaxing lending condition into account, I gradually relax the incentive constraint until period 7. From period 8 on, θ^I goes back to the steady states.

Figure 12 plots the response paths of key endogenous variables. Income dynamics is the main force driving household consumption, leading to the hump-shaped response. In calibration, the borrower household is assumed more impatient and less risk-averse than the depositor household. The former is modeled after the young generation, while the latter is modeled after the old one. The simulation shows that the borrower's consumption is more volatile than depositor's, mainly driven by the difference in risk attitude.

⁴¹ Because the borrower household cannot save, I use the risk-free rate at steady state $R_{b,ss}$ to define the model counterpart of the mortgage spread.

⁴² I use the stationary component of GDP per capita estimated in Section 5.1.3. The GDP per capita in 2001 is normalized to 1 to be consistent with the start of the boom-bust simulation from the steady state.

House and Mortgage Prices. While the dynamic of house price p is driven by the income process in the boom episode, it is mainly attributed to the enduring impact of increasing unitized risk instead of the negative income shocks in the bust episode. The decrease in the house price is more than the increase in the run-up of the crisis. The asymmetric response is attributed to the foreclosure loss and persistence of unitized risk that create amplified and prolonged responses of the house price in the bust periods.

The mortgage price q consists of two components in the pricing equation (3.11), the shadow cost of borrowing λ^B and the mortgage continuation value. With long-term defaultable mortgage, the continuation value takes into account the repayment probability π and the future mortgage price. In the housing boom, higher income mitigates the borrowing constraint (λ^B decreases). The sequence of positive income shocks eases the borrower's tradeoff between frontloading consumption and incurring higher shadow cost. The increase of the continuation value of mortgage is offset by the decrease in the shadow cost of borrowing, explaining limited response of the mortgage price in the boom periods. When the housing bust comes, the mortgage price plummets deep to reflect the decrease of mortgage continuation value. Even though the tightening of the borrowing constraint is short lived, the recovery of the mortgage price is lengthy due to the enduring impact of mortgage default.

New Mortgage Issuance and Default Rate. With long-term mortgage, the stock and the flow of debt differ; new mortgage issuance ql^B is more responsive than the debt outstanding qm^B . A series of positive income shocks induces the borrower household to frontload consumption by borrowing more and to relax the borrowing constraint. Even when income growth reaches the peak in 2004, the growth of new mortgage issuance is sustained and fueled by increasing mortgage risk until the bust in 2008. The model predicts a 20% sizable decrease in new mortgage issuance from the boom to the bust.

Besides borrowing new debt, an alternative way to manage the mortgage stock is through the default channel. Both borrowing less new debt and defaulting more old debt reduce the mortgage outstanding. The default rate in the boom periods is relatively flat, because two countervailing forces are balanced. The series of positive income shocks push the default rate downward, while the increasing mortgage risk exerts upward pressure on the default rate. As unitized risk climbs up from 2004, the share of new issuance in the mortgage stock increases. It indicates that mortgage default is to rise and to retire more old debt, and that the mortgage structure shifts toward more short-term borrowing. I find the loosening credit condition has limited explanatory power in the dynamics of the mortgage stock and flow.

Interest Rate and Spread. The dynamic of the implied mortgage rate R_q is governed by the income in the boom and by the unitized risk in the bust. The implied mortgage rate measures the *ex ante* cost of mortgage and is counter-cyclical. R_q decreases in the housing boom, because expected default is low and mortgage value is expected high. In the bust periods, mortgage default is high and enduring, leading to the increase and lengthy recovery of the implied mortgage rate. The risky return R_m is an *ex post* measure. It increases in the boom, but decreases in the bust due to rising default loss. The movement of the risk-free rate R_b is attributed to the aggregate income and the relaxation of the credit lending condition.

In Table 9, I report the contributing factors in the dynamics of the excess return $R_m - R_{b,-1}$ and the mortgage spread $R_q - R_{b,ss}$. Compared with the data, the timing and the size of changes are plausible overall. The downward trend of the excess return $R_m - R_{b,-1}$ in boom episode is mostly related to the loosening lending condition, while both income and lending condition contribute to the spike of the excess return in the bust episode. Income dynamic gives rise to limited downward pressure to excess return in the

housing boom, so it is insufficient to explain the change in excess return by itself. The mortgage spread $R_q - R_{b,ss}$ is mainly attributed to income in the boom and to unitized risk in the bust, with limited role of lending condition in the dynamic.

Leverage. The growth of the intermediary's leverage ϕ in the boom is attributed to the lending relaxation. If there were no growth of deposit b , more asset due to an increasing propensity to borrow would imply a decrease in the leverage. However, a series of positive income shocks induces the depositor household to backload consumption by saving more, leading to an increase in an intermediary's leverage. As lending relaxation indicates an increase in an intermediary's power to arbitrage away the excess return, more mortgage credit can be channeled from the depositor to the borrower, thus pushing up the leverage in the boom.

In the counterfactual experiment with the income shock alone, the leverage amplifies the hike of the excess return in the bust episode. The reason is that the excess return is assigned a smaller weight Ω in the bust episode due to intermediary's deleverage. This creates asymmetric response of the excess return to the income shocks.

Borrowing vs Lending Constraint. The dynamics of the multipliers λ^B and ν_m show that the credit constraints faced by the demand and the supply side are mitigated in the boom and tightened in the bust. The counterfactual experiments, however, indicate different sources of shocks contributes to their dynamics. The path of borrower's multiplier λ^B is explained by the income shocks, while the path of intermediary's multiplier ν_m is attributed to the credit lending condition. In fact, with positive income shocks only, the lending constraint will be slightly tightened in the boom episode. The reason is that the growth of R_m due to excess credit demand pushes up ν_m , which is illustrated in (4.1). Hence, relaxation of the credit lending condition is essential to mitigate the credit constraints of the supply side and to generate sufficient downward pressure to the excess return.

6.3 Intermediary Asset Pricing

The experiment in Section 6.2 shows that lending condition is crucial to explain the downward trend of the excess return in the boom episode, while this section emphasizes how a financial intermediary amplifies and propagates aggregate shocks in the boom-bust episode. The moral hazard problem of an intermediary imposes the incentive constraint and motivates an endogenous leverage $\phi = \nu_n/(\theta^I - \nu_m)$ that is pro-cyclical: high (low) marginal values of net worth and asset induce an intermediary to increase (decrease) leverage in the boom (bust) episode.

To evaluate the impact of pro-cyclical leverage in a comparable setting, I consider an alternative economy where an intermediary's leverage is exogenous.

$$\phi^{NF}(S) = \frac{\nu_{n,ss}}{\theta^I(S) - \nu_{m,ss}}$$

Thus, the exogenous leverage cannot react to the endogenous, but the exogenous states of the economy. If lending condition shock is expected to be constant from now on, the exogenous leverage is constant and intermediary's pricing kernel is degenerate to the depositor's stochastic discount factor (see 3.21). The alternative economy thus minimizes intermediary's impact by allowing the depositor to price the risky asset directly, but is still comparable to the benchmark by keeping the constraint on credit supply. Figure 13 shows the dynamics of excess return and leverage in the benchmark and the alternative boom-bust simulations.

With pro-cyclical marginal values of net worth and asset, the increase of the benchmark leverage is about five times bigger than the increase of the exogenous leverage in the boom episode. Furthermore, pro-cyclical leverage amplifies the response of excess return by more than three times, generating stronger counter-cyclical variation in the mortgage spread.⁴³

The pro-cyclical augmented term $\Omega(S')$ contributes to the amplification of excess return in intermediary asset pricing. Two assumptions are crucial, (1) an intermediary is long-lived, $\psi > 0$; (2) the share of asset diversion is sufficiently large, $\theta^I > 0$. The first assumption implies that amplification comes from the continuation value of an intermediary. The second assumption emphasizes the role of agency friction, so depositor's and intermediary's marginal valuation sufficiently differs across states. If either assumption fails ($\psi = 0$ or $\theta^I = 0$), the term Ω' will be constant for all future states.

In the continuation value of an intermediary, what $\Omega(S')$ captures is the future optimal leverage ϕ' , and thus the future marginal value of net worth and asset ν'_n and ν'_m . As is discussed in Section 3.3, ν_m as an indicator of the tightness of lending constraint measures liquidity premium, the intermediary's ability to arbitrage away the excess return. The term $\Omega(S')$ implies that when an intermediary prices current net worth and asset, the belief of future liquidity premium directly matters. As a result, it establishes a direct link of current and future optimal leverages. For illustration, consider the benchmark model with perfect foresight. Using the optimality conditions (3.21) and (3.26), we can rewrite intermediary's leverage as follows.

$$\phi(S) = \frac{\nu_n(S)}{\theta^I - \nu_m(S)} = \frac{\Omega(\mathbf{K}(S))}{\theta^I(S) - \Omega(\mathbf{K}(S))\Lambda_D(S, \mathbf{K}(S))[R_m(S, \mathbf{K}(S)) - R_b(S)]} \quad (6.1)$$

where $\Omega(S) = 1 - \psi + \psi\theta^I(S)\phi(S)$

To illustrate the impact of direct serial correlation, consider an unexpected one-shot positive deviation of leverage from the steady state. If there is no direct serial correlation, the leverage will monotonically converge to the steady state. However, when current optimal leverage puts weight on future leverage, its response path will be hump-shaped. It is suboptimal to immediately force the leverage back to the steady state, because extending the positive deviation of leverage increases the marginal value of net worth ν_n and thus the present discounted value of an intermediary. The generalized impulse responses to the aggregate shocks (Figures 14, 15, 16) show that the benchmark case with intermediary asset pricing generates larger and more volatile response of excess return than the case in which an intermediary cannot manage the balance sheet but adopt an exogenous leverage.

6.4 Why Mortgage Risk Increased, But Mortgage Spread Decreased?

The section uses the calibrated model to quantify the time-varying component that contributes to the decreasing mortgage spread. To examine the mortgage spread in the model, I focus on the expected excess return (EER) that is defined as the spread of the expected asset return over the risk-free return. Using intermediary's optimality condition (3.21), EER can be written as the sum of two components, the liquidity

⁴³ The simulated paths of other endogenous variables are not so different in two simulations.

premium and the default premium.

$$\begin{aligned}
\underbrace{E[R_m(S, S') - R_b(S)]}_{\text{EER}} &= \underbrace{\frac{\nu_m(S)}{\nu_n(S)} R_b(S)}_{\text{Liquidity Premium}} - \underbrace{\frac{\text{Cov}[\Lambda_D(S, S') \Omega(S'), R_m(S, S')]}{\nu_n(S)}}_{\text{Default Premium}} R_b(S) \\
&\text{where } \Omega(S') = 1 - \psi + \psi \theta^I(S') \phi(S') \text{ and } \phi(S') = \frac{\nu_n(S')}{\theta^I(S') - \nu_m(S')}
\end{aligned} \tag{6.2}$$

The liquidity and the default premiums depend on the current and future shadow costs of credit supply $\nu_m(S)$ and $\nu_m(S')$, respectively. Thus, any bad news on intermediary's current or future funding liquidity will tighten the lending constraint and push up the mortgage spread.

Suppose there is one-shot reduction of the start-up transfer κ . The unexpected decrease in the net worth will tighten the lending constraint. The liquidity premium which measures intermediary's ability to arbitrage away the excess return will go up and increase the mortgage spread. Bad news about future net worth is factored into current EER through the augmented term $\Omega(S')$ in the default premium. Suppose there is an unexpected one-shot increase in the mortgage risk σ_ω . The news is revealed at the end of current period, but before the future shocks are realized.⁴⁴ There are two channels through which the default premium will respond to the news. First, the default premium will be bigger, because the depositor cannot reduce risk-free lending at news release, forced to undertake additional volatility in consumption across future states. Second, agency friction will amplify the first channel through intermediary asset pricing.⁴⁵

To understand how important two premiums are and how they evolve in the boom-bust episode, I examine their relative contribution in EER instead of the realized excess return simulated in Figure 12. EER is forward-looking, while the realized excess return is backward-looking. I decompose EER based on the following simulation. Starting from the stochastic steady state, I project the realized path of the exogenous processes to follow the simulated path in Figure 11. At each period, the next-period shocks are simulated in order to calculate the expected asset return $R_m(S, S')$ and other intertemporal terms. The premium components are defined as the ensemble means of the simulated values.

Figure 17 shows the decomposition of EER into liquidity and default premiums. EER follows a similar path as the realized excess return, although they are essentially different. At the stochastic steady state, 5.5% of EER is default premium and the rest 94.5% is liquidity premium. The decrease of EER in the boom episode stems from the decrease of liquidity premium. Funding liquidity need is mitigated by 87% at the end of boom, while the default premium increases from 13 to 30 basis points as mortgage risk increases. Before the bust, the default premium weighs similarly as liquidity premium in EER, accounting to 51% of EER. The immediate jump of EER is driven by the surge of liquidity need. The default premium goes back to 13 basis points as mortgage risk ebbs back to normal in the bust episode.

The decomposition of EER shows that the macro condition of credit supply is strongly time-varying and cannot simply be assumed constant in the housing boom. The increasing default premium in the boom is consistent with increasing mortgage risk that captures the time-varying macro condition of credit demand, but is dominated by decreasing liquidity premium.

⁴⁴ The liquidity premium won't be updated in the experiment, because decisions in the current period are already made.

⁴⁵ Concretely, if the asset is expected riskier, the depositor will reduce risk-free lending in the next period. The net worth of both continuing and start-up intermediaries is expected to decrease faster than the mortgage balance, which tightens the future incentive constraint and implies a higher leverage in the next period. Intermediary's marginal valuation $\Lambda_D(S, S')$ which is higher than the steady state in the case of poor asset return will be magnified by a higher $\Omega(S')$ due to the increase in future leverage. Additionally, the future asset return will be lower, because of higher default rate. Those two changes lead to a more negative covariance term. The default premium thus takes into account both the negative correlation between marginal valuation and the asset return as well as the agency friction.

6.5 Model Sensitivity

The section conducts sensitivity analysis on the model assumptions. Specifically, I examine two assumptions on the mortgage: default option and long-term contract. For comparison to the benchmark, I consider two alternative economies, one with non-defaultable mortgage and the other with short-term debt. Figures 14, 15 and 16 show the generalized impulse response to +1 standard deviation (SD) of income shock y , to +1 SD of unitized risk σ_ω and to -1 SD of divertible share θ^I , respectively. Besides the benchmark case, the impulse responses in two alternative economies are reported.

Default Option. To examine the role of default option, I consider an economy with full commitment where the borrower household cannot make the state-dependent default choice. Instead, a constant share δ of mortgage will go default in each period. In the benchmark, the borrower can deleverage either by borrowing less new debt or by defaulting more old debt. In the counterfactual with non-defaultable mortgage, the borrower is forced to borrow less in response to the negative income, which boosts the response of new borrowing by 30%.⁴⁶

Two ways of deleverage are not perfectly substitutable and incur different costs. When hit by a single and small negative shock, the borrower finds it less costly to reduce new borrowing than to increase the default rate. Mortgage default in the bust when multiple negative shocks arrive serves to mitigate the deleveraging pressure and to smooth the borrower's consumption *ex post* across the aggregate states. Keeping the default rate low unless the aggregate condition is bad enough is optimal, because too much *ex post* default will worsen the *ex ante* borrowing condition through a lower mortgage price q and thus a higher implied mortgage rate R_q . Hence, the paths with and without the default option in response to a single or small shock are similar due to small movement of the default rate, while the default option is crucial in the boom-bust episode with multiple and large shocks to help the household to deleverage.

Long-Term Contract. I compare the benchmark to an alternative economy with the short-term defaultable mortgage, which is implemented by $\mu = 1 - \varphi_0$ and $\rho = 0$. With the short-term contract, mortgage stock qm^B and flow ql^B coincide, making the former more responsive to the shocks. As a consequence, short-term contract results in the convergence of prices and allocation to the steady state at a faster rate, taking about 50% of the time in the benchmark for the initial response to decay by half. Hence, persistence introduced by the long-term debt is essential to explain the slow reduction of mortgage balance and the plummet of new borrowing in the bust episode.

Difference between long-term and short-term mortgages also lies in the extent of intertemporal consumption smoothing. With the short-term mortgage, a borrower who decides not to default in the next period has to pay off the debt rather than to smoothly repay the balance over an infinite horizon. As a result, the level of the default rate $1 - \pi$ and the implied mortgage rate R_q with short-term debt are much higher (at 17% and 10% respectively). At the steady state before any shock is realized, if the borrower were given the choice to choose whether to live in a world with short-term mortgage or a world with long-term mortgage, she would prefer the latter. The additional welfare benefit brought by the long-term contract is equivalent to 2.87% of borrower's permanent consumption.⁴⁷

⁴⁶ Because unitized risk takes effect through the mortgage default channel, turning off the default option will completely shut down the response to the unitized shock.

⁴⁷ The welfare benefit ι is defined as

$$V_{ss,L}^B = \frac{u(c_{ss,L}^B, h^B)}{1 - \beta_B} = \frac{u((1 + \iota)c_{ss,S}^B, h^B)}{1 - \beta_B} = (1 + \iota)^{(1-\alpha)(1-\xi^B)} V_{ss,S}^B$$

7 Conclusion

In this paper, I document a mortgage market puzzle in the run-up of the Great Recession: the mortgage risk increased while the mortgage spread decreased. The evidence suggests that, besides the mortgage risk component, another time-varying factors decreased the mortgage spread in the boom episode. I develop a general equilibrium model with borrowers, depositors and intermediaries in a dynamic economy with multiple aggregate risks. Besides the income shock, I introduce and calibrate the mortgage risk and the lending condition as the aggregate risk to capture the time-varying macro conditions of the credit demand and supply. The paper adds to the credit supply view that emphasizes the role of the lending relaxation of intermediaries in the credit boom episode. I distinguish the impact of the lending relaxation from that of the borrowing relaxation. I show that the former is consistent with the trend of the mortgage credit and the mortgage spread, while the latter is not.

I simulate the boom-bust episodes and quantify the contribution of the income, the mortgage risk and the lending condition as the aggregate risks to the mortgage market dynamics. The income channel alone cannot generate the observed movement in the mortgage spread under plausible sizes of shocks. I highlight that the mortgage risk and the lending condition were time-varying in the boom episode and contributed to the trend of the mortgage credit and the mortgage spread in the period. The lending relaxation explains the downward movement of the excess return in the boom, while the mortgage risk accounts for the spike of the mortgage spread in the bust. By decomposing the mortgage spread into a liquidity premium and a default premium, I show that the increase in the default premium that reflected the growing mortgage risk was outweighed by a faster decrease in the liquidity premium, leading the mortgage spread to decrease in the run-up of the Great Recession.

$V_{ss,L}^B$ and $V_{ss,S}^B$ are borrower's steady state values with long-term and short-term mortgage, respectively.

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A Data and Empirics

I present the time trend from the demand and the supply side of the US mortgage market, and attempt to find clues from micro data to complement macro evidence. Micro data, on one hand, are robust to aggregation in the presence of heterogeneity. The drawback, however, is that micro data representative of the whole market are hard to obtain and to be linked to other aspects of the macro economy. Macro data, on the other hand, summarize the market as a whole, but harder to be linked to individual behaviors. The difference will mainly lie in the aggregation method in calculating the empirical counterparts of model-derived ratios and statistics.

Figure 1a plots the trend of mortgage-to-GDP ratio and mortgage-to-disposable income ratio. Mortgage refers to the home mortgage liability in the category of Households and Nonprofit Organizations from Board of Governors of the Federal Reserve System, Z.1 Table [FL153165105.Q] and FRED [HMLBSHNO]. GDP is defined as the nominal US gross domestic product from US Bureau of Economic Analysis [A191RC] and FRED [GDP]. The disposable income is from US Bureau of Economic Analysis [A067RC] and FRED [DSPI]. Figure 1b shows the time trend of mortgage-to-Real Estate ratio. Real estate value refers to the total market value of real estate, and is obtained from Board of Governors of the Federal Reserve System, S.3.q Table [FL155035005.Q] and FRED [HNOREMQ027S].

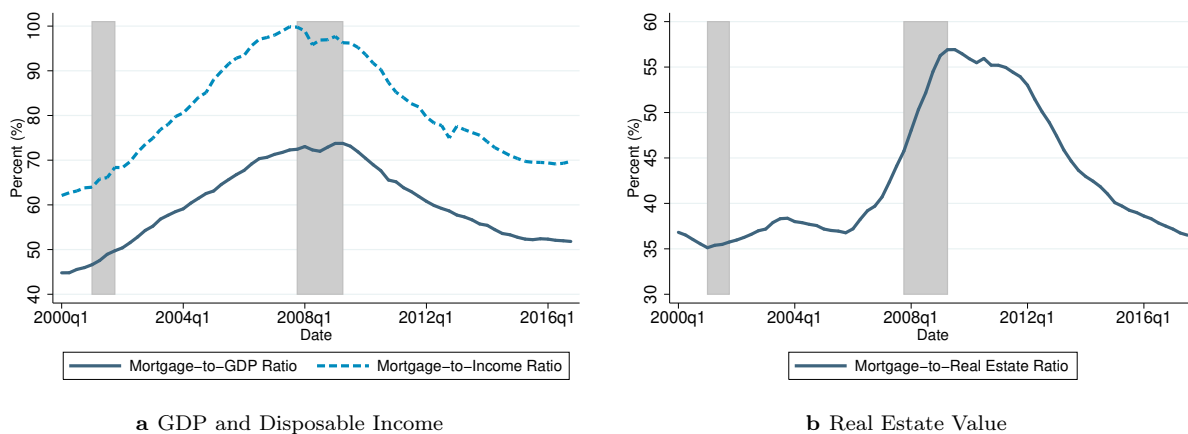


Figure 1: Ratios of mortgage outstanding to GDP, disposable income and real estate value. Source: Board of Governors, BEA.

Figure 2 shows the time trend of US real residential housing price index (normalized to 100 in year 2010) and its growth rate. The data is obtained the table of Residential property prices: selected series (nominal and real) from Bank for International Settlements [Q:US:R:628] and FRED [QUSR628BIS].

Figure 3 shows the trend of the share of agency mortgage backed securities (Fannie Mae, Freddie Mac, Ginnie Mae) in the US household mortgage outstanding on left axis, and the time trend of the ratio of agency MBS in total MBS outstanding on the right axis. The source of household mortgage outstanding is defined in Figure 1a, while the total agency MBS outstanding is summarized in Board of Governors of the Federal Reserve System, 1.54 Table.⁴⁸ Total MBS is defined as the total mortgage outstanding held by mortgage pool or trusts, including both agency or private conduit loans, in 1.54 Table.

⁴⁸ During the Great Recession, the GSEs were under conservatorship. Part of the securitized loans were held by the federal agencies through Large-Scale Asset Purchases (also known as Quantitative Easing) starting from November, 2008. I define the agency loans here to include those under the category of both “Federal agency” and “Mortgage Pool or Trusts”.

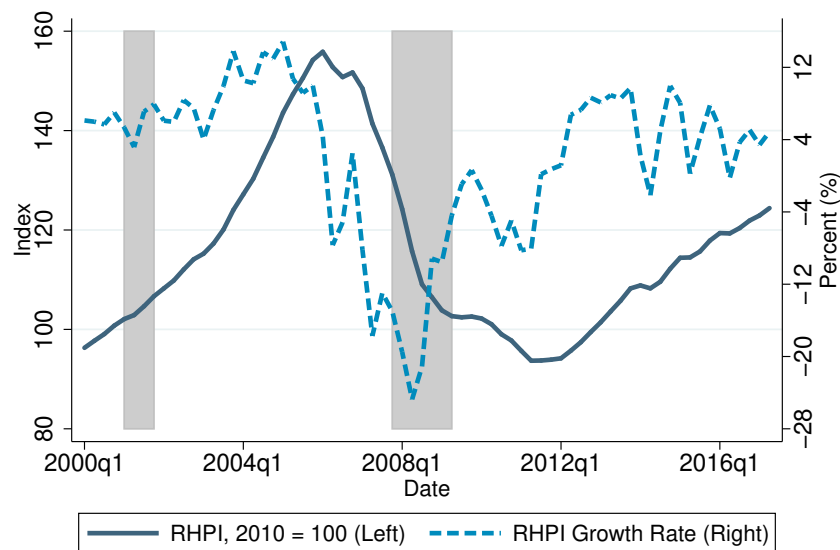


Figure 2: Real housing price and growth rate. Source: BIS.

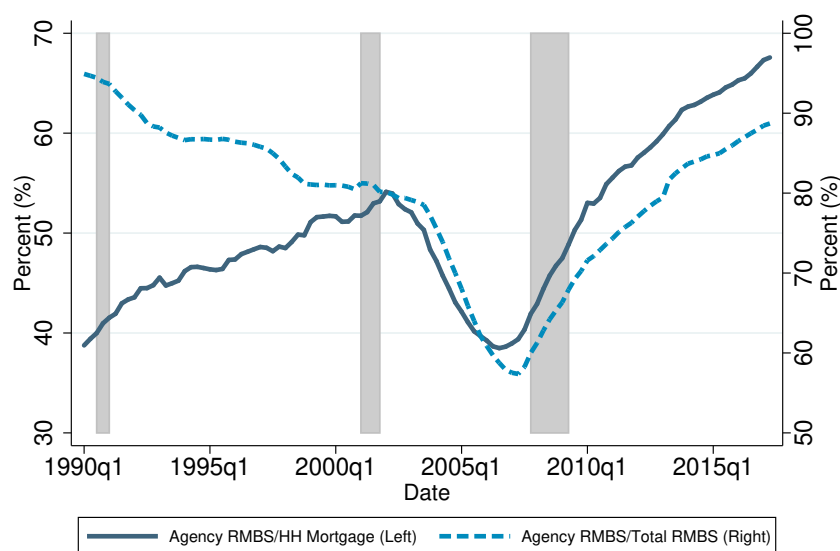


Figure 3: Share of agency mortgage backed securities. Source: Board of Governors.

As to the trend of non-agency securitized loans, I use the Columbia Collateral Files generated by Wells Fargo Bank that serves as the trustee for the private-label mortgage backed securities (PLMBS). By the beginning of January, 2008, The database covers more than 4 million securitized loans. Although the loans in the database account to approximately 10 percent of the PLMBS market share, Wells Fargo Bank was listed as the top retail mortgage lender and servicer nationwide (Inside Mortgage Finance, 2012). The sample universe covers all types of riskiness (prime, Alt-A and subprime) and mortgage products (adjustable rate mortgage and fixed rate mortgage). Hence, the summary statistics, to some extent, are informative about the PLMBS market. The database includes monthly cross-section starting from December, 2006. After 2008, the PLMBS market shrank dramatically (Figure 3). Thus, the micro evidence presented from the Columbia

Collateral Files focus on the loans originated before the 2008 Great Recession.⁴⁹

Table 1 and 2 report the number of mortgage loans and dollar volume by each mortgage product and origination year. The adjustable rate mortgages are further classified, depending on whether an ARM loan is a balloon, an interest-only, or a negative-amortized loan. As three characteristics are not mutually exclusive, there are 8 possibilities in total. The fixed rate mortgages are classified, depending on whether a FRM loan is fully amortized.

Table 1: Mortgage Frequency by Product Types and Origination Year

	Origination Year													
	2001 No.	Col %	2002 No.	Col %	2003 No.	Col %	2004 No.	Col %	2005 No.	Col %	2006 No.	Col %	2007 No.	Col %
ARM														
!B, IO, NegAm	0	0.00	6	0.03	166	0.15	454	0.14	383	0.06	21,228	2.34	17,801	7.50
!B, IO,!NegAm	282	6.04	2,649	12.41	21,620	19.30	101,265	31.56	202,668	31.32	195,635	21.55	44,745	18.85
!B,!IO, NegAm	93	1.99	361	1.69	1,474	1.32	25,928	8.08	70,653	10.92	85,697	9.44	35,156	14.81
!B,!IO,!NegAm	2,200	47.13	9,286	43.49	33,878	30.24	103,634	32.29	180,586	27.90	172,347	18.98	39,129	16.48
B, IO, NegAm	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	5	0.00	90	0.04
B, IO,!NegAm	0	0.00	0	0.00	338	0.30	1,202	0.37	2,394	0.37	12,589	1.39	1,679	0.71
B,!IO, NegAm	0	0.00	0	0.00	2	0.00	1	0.00	95	0.01	11,351	1.25	427	0.18
B,!IO,!NegAm	20	0.43	157	0.74	309	0.28	2,599	0.81	14,836	2.29	102,189	11.25	23,596	9.94
Total	2,595	55.59	12,459	58.35	57,787	51.58	235,083	73.26	471,615	72.88	601,041	66.19	162,623	68.50
FRM														
B	106	2.27	252	1.18	1,189	1.06	6,372	1.99	36,774	5.68	78,661	8.66	10,751	4.53
F	1,967	42.14	8,641	40.47	53,062	47.36	79,446	24.76	138,764	21.44	228,296	25.14	64,037	26.97
Total	2,073	44.41	8,893	41.65	54,251	48.42	85,818	26.74	175,538	27.12	306,957	33.81	74,788	31.50

Note: ARM = adjustable rate mortgage. FRM = fixed rate mortgage. B = Balloon. IO = Interest-Only loans. NegAm = Negative Amortization. The exclamation mark (!) refers to negation. Source: Columbia Collateral Files from Wells Fargo and author's calculation.

Table 2: Mortgage Dollar Volume by Product Types and Origination Year

	Origination Year													
	2001 million	Col %	2002 million	Col %	2003 million	Col %	2004 million	Col %	2005 million	Col %	2006 million	Col %	2007 million	Col %
ARM														
!B, IO, NegAm	0	0.00	2.45	0.05	38.69	0.12	145.77	0.19	145.57	0.09	7,954	3.24	6,808	8.29
!B, IO,!NegAm	114.63	13.37	974.57	18.56	8,249	25.45	33,100	43.54	75,212	44.21	77,118	31.39	22,009	26.80
!B,!IO, NegAm	10.69	1.25	56.91	1.08	287.01	0.89	5,852	7.70	20,867	12.27	32,202	13.11	13,850	16.86
!B,!IO,!NegAm	377.36	44.03	1,640	31.23	8,755	27.02	21,581	28.39	39,476	23.21	37,824	15.40	10,867	13.23
B, IO, NegAm	0	0.00	0	0.00	0	0.00	0	0.00	0	0.00	1.8	0.00	34.82	0.04
B, IO,!NegAm	0	0.00	0	0.00	71.94	0.22	278.97	0.37	689.5	0.41	3,810	1.55	530.56	0.65
B,!IO, NegAm	0	0.00	0	0.00	0.71	0.00	0.19	0.00	34.18	0.02	4,404	1.79	179.66	0.22
B,!IO,!NegAm	3.19	0.37	17.86	0.34	71.8	0.22	359.2	0.47	4,006	2.35	26,204	10.67	6,791	8.27
Total	505.87	59.02	2,691	51.26	17,474	53.92	61,318	80.66	140,431	82.55	189,519	77.14	61,070	74.35
FRM														
B	8.5	0.99	23.65	0.45	109.97	0.34	317.48	0.42	2,450	1.44	7,124	2.90	2,027	2.47
F	342.71	39.99	2,535	48.29	14,824	45.74	14,386	18.92	27,238	16.01	49,026	19.96	19,039	23.18
Total	351.21	40.98	2,559	48.74	14,934	46.08	14,704	19.34	29,689	17.45	56,151	22.86	21,066	25.65

Note: ARM = adjustable rate mortgage. FRM = fixed rate mortgage. B = balloon loans. IO = interest-only loans. NegAm = negative amortization loans. The exclamation mark (!) refers to negation. Dollar volume of each origination year is measured by the sum of original balance. Source: Columbia Collateral Files from Wells Fargo and author's calculation.

Figure 4a shows the time trend of foreclosure rate, and (serious) delinquency rate. The delinquency rate refers to the percentage of single-family residential mortgages from the domestic offices of all commercial banks that are past due 30 days or more and still accruing interest as well as those in non-accrual status. The data is obtained from the table of Charge-Off and Delinquency Rates on Loans and Leases at Commercial Banks from Board of Governors of the Federal Reserve System, and FRED [DRSFRMACBS]. The serious

⁴⁹ The loans used for aggregation here come from a subset in which loans have complete information on origination date between 2001 and 2007, original balance, FICO score, loan-to-value ratio, documentation type, appraisal value. Approximately 2.5 million observations are used.

delinquency rate is defined as the percentage of loans that are past due 90 days or more. The foreclosure rate is defined as the percentage of mortgages in the foreclosure process. Both the serious delinquency rate and foreclosure rate are calculated using securitized loan data from CoreLogic Loan Performance database. Figure 4b reports the share of non-agency securitized mortgages for each vintage (origination) year that were current by January 1, 2008, but were past due 60 days or more at least once in 2008. The delinquency shares are derived from the monthly payment history documented in Columbia Collateral Files from Wells Fargo.

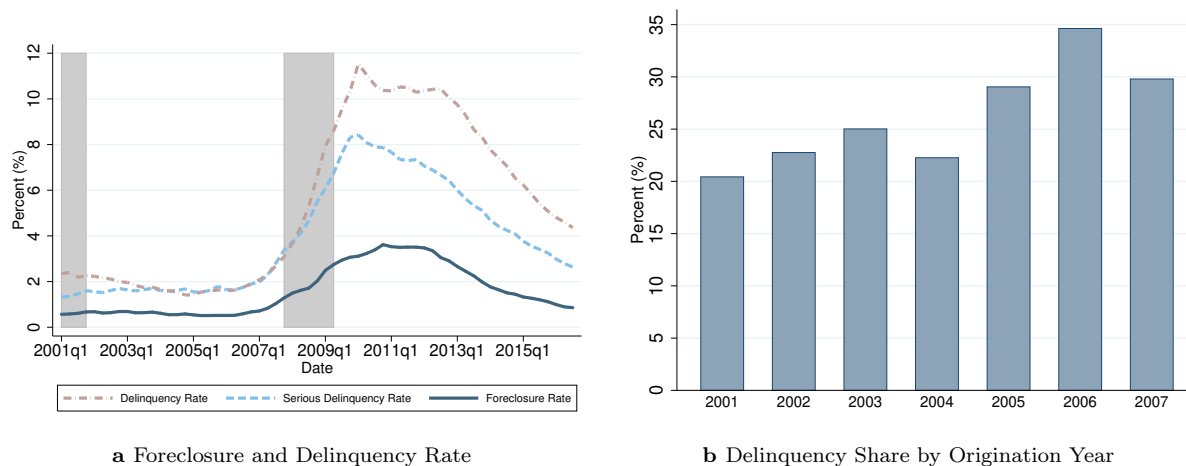


Figure 4: Default rate and non-agency loan performance in 2008 by origination year. Source: CoreLogic, Board of Governors, Columbia Collateral Files from Wells Fargo.

Figure 5a shows the time trend of the average FICO score by mortgage product (fixed rate or the adjustable rate mortgages) and origination, which is a standardized number between 300 and 850 to evaluate the credit worthiness of a mortgagor. The higher the score, the more likely a mortgagor will repay the debt.⁵⁰ Figure 5b presents the loan-to-value ratio of private-label securitized loans mortgage product and origination. Figure 5c shows the trend of the share of three documentation type at mortgage application by origination: full, limited, or stated documents. Each type have different level of verifiable information on income, assets and employment. Figure 5d show the mortgage spread of fixed rate mortgages over the 10-Year Constant Maturity Treasury Rate, and the gross margin of adjustable rate mortgages which is the mortgage rate deducted by the floating index rate. Both gross margin and index rate are specified at the beginning of a mortgage contract.

Figure 6 shows the time trend of macro and micro bank leverage of the US banks. The macro leverage is the ratio of total bank asset to total bank equity (Figure 6a). Two data sources are used to calculate the macro leverage, Compustat Capital IQ and FFIEC. Compustat data covers the bank-level balance sheet data from 1962, while FFIEC reports the aggregate balance sheet data of all US commercial banks on total asset from 1973 [TLAACBM027NBOG] and total equity from 1984 [USTEQC]. The total asset and total equity are log-detrended before taking the ratios. The micro leverage ratios are the inverse of the median equity-to-asset ratio and the median Tier-1 risk adjusted capital ratio (Figure 6b). The quarterly equity-to-asset ratio is calculated by pooling the banks in the US that were active in that quarter and had a positive value of a bank's total asset, and by taking the median of the ratio of the total equity [SEQQ] (or tier

⁵⁰ The most widely used credit scores are FICO scores created by Fair Isaac Corporation. For the details on FICO scores, see <https://www.myfico.com/credit-education/credit-scores/>.



Figure 5: Trend of borrower and loan characteristics and mortgage spreads by origination year. Source: Columbia Collateral Files from Wells Fargo.

1 capital) and total asset [ATQ] from the balance sheet. The equity of banks with missing equity values is backed out from total liability [LTQ] if debt information is available. The risk adjusted capital ratio (tier 1) [CAPR1Q] is defined as the tier 1 equity as a percent of adjusted risk-weighted assets. The risk adjustment of assets takes into account both on- and off-balance sheet items and assign higher weight to riskier assets. 1993q1 is the first quarter to document the risk adjusted capital ratio, because the calculation guidance was finalized in 1992. The risk adjusted capital ratio with tier 1 and tier 2 capital combined has a similar time series pattern. The bank-level data is obtained from Compustat Bank Fundamentals Quarterly through Wharton Research Data Services (WRDS). Refer to Compustat Data Definition Chapter for details http://web.utk.edu/~prdaves/Computerhelp/COMPUSTAT/Compustat_manuals/user_05r.pdf.

Figure 7a plots the time trend of US 30-year Fixed Rate Mortgage Rate, and its spread with respect to US 10-year Constant Maturity Treasury (CMT) Rate. The long-term instead of the short term risk-free rate is used to adjust the term premium, because the household mortgage is relatively illiquid compared with the mortgage backed security. The FRM rate data is obtained from Primary Mortgage Market Survey from Freddie Mac and FRED [MORTGAGE30US]. The CMT data is from Board of Governors of the Federal Reserve System, H.15 Table and FRED [DGS7].

Figure 7b shows the time trend of the MBS spread defined as the annualized total return associated with

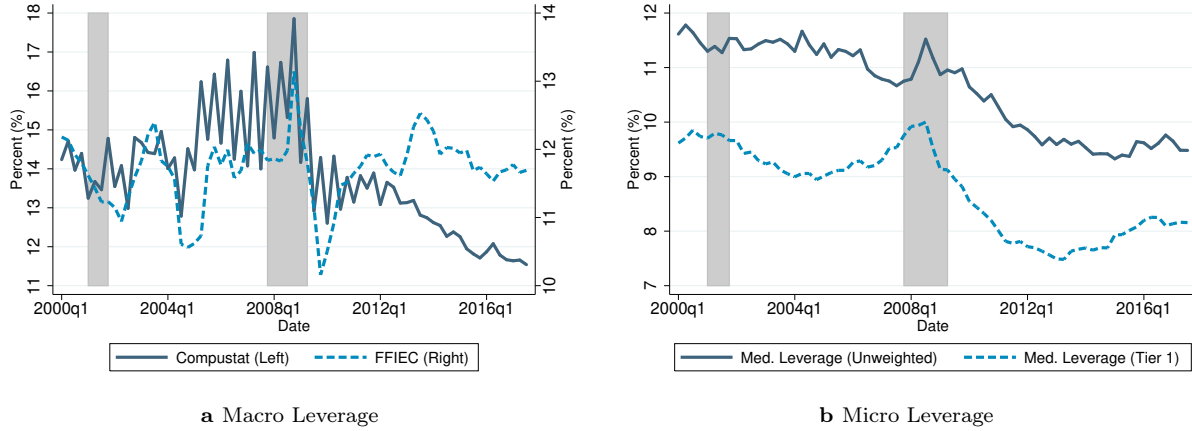


Figure 6: Macro and micro leverage, US Banks. Source: Compustat Capital IQ, FFIEC.

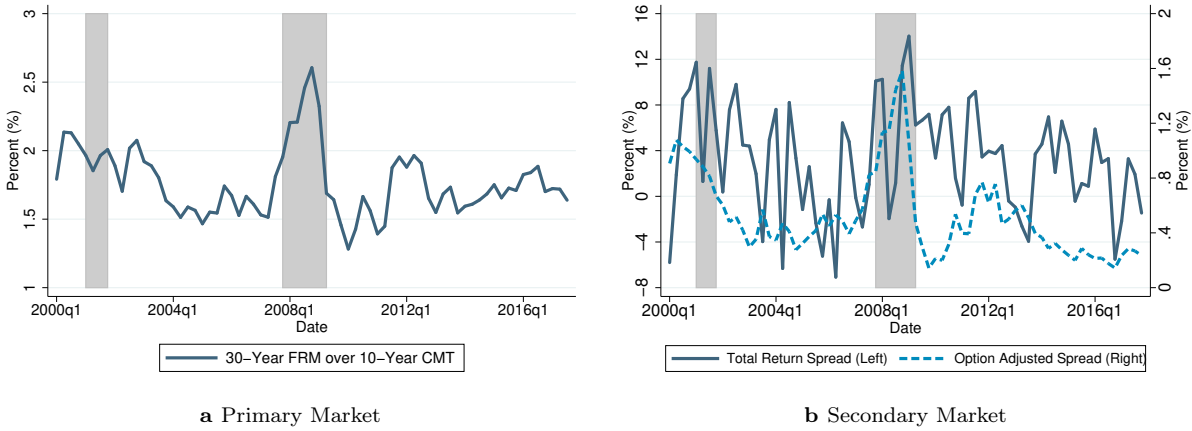


Figure 7: Agency mortgage spread in the primary and secondary mortgage markets. Source: Freddie Mac, Board of Governors, Barclays, Bloomberg.

Barclays MBS Index over 1-Year Constant Maturity Treasury Rate.⁵¹ The Barclays MBS Price Index is designed to measure the performance of the US agency mortgage pass-through segment of the US investment grade bond market.⁵² The Index which documents the longest history of US agency-backed securities is obtained from the Bloomberg. Empirically, the option-adjusted spread (OAS) is the commonly used MBS spread in the presence of prepayment risks. OAS which is based on a Monte Carlo simulation model, takes into account the interest rate and prepayment risks through a presumed prepayment model. Figure 7b plots the trend of option adjusted spread.

⁵¹ Total return consists of price return, coupon return, paydown return, currency return, writedown return prepayment premium return. Refer to Bloomberg Barclays Index Methodology <https://www.bbhub.io/indices/sites/2/2017/03/Index-Methodology-2017-03-17-FINAL-FINAL.pdf>.

⁵² It refers to a category of pass-through securities backed by pools of mortgages and issued by the following Government Sponsored Enterprises (GSE): Government National Mortgage Association (GNMA, or Ginnie Mae), Federal National Mortgage Association (FNMA, or Fannie Mae) or Federal Home Loan Mortgage Corporation (FHLMC, or Freddie Mac).

B Equilibrium Conditions and Steady State

B.1 Equilibrium Conditions

Denote the state variables $S = [m_{-1}, \hat{b}_{-1}, S^{exo}]$ where m_{-1} and $\hat{b}_{-1}$ are the begin-of-period mortgage coupon of the borrower and the deposit repayment from the financial intermediary, respectively. $S^{exo} = [y, \sigma_\omega, \theta^I]$ is the exogenous state variables. Together with the market clearing conditions, the controls $\mathcal{C}(S) = [c^B(S), c^D(S), \nu_m(S), \nu_n(S), \lambda(S), p(S), q(S), R_b(S)]$ solve the following functional equations.

$$q(S) - \lambda(S)\varphi = E \Lambda_B(S, S') \pi(\omega^*(S')) [(1 - \rho)\varphi + \rho(1 + \mu q(S'))] \quad (\text{B.1})$$

$$\text{where } w^*(S) = \frac{[(1 - \rho)\varphi + \rho(1 + \mu q(S))]m_{-1}}{p(S)\bar{h}^B}, \quad \Lambda_i(S, S') = \beta_i \left(\frac{c^i(S)}{c^i(S')} \right)^{(1-\alpha)\xi + \alpha}$$

$$[1 - \theta^B \lambda(S)]p(S) = \frac{\alpha}{1 - \alpha} \frac{c^B(S)}{\bar{h}^B} + E \Lambda_B(S, S') p(S') z_1(\omega^*(S')) \quad (\text{B.2})$$

$$0 = \lambda(S)[\theta^B p(S)\bar{h}^B - \varphi m(S)] \quad (\text{B.3})$$

$$\nu_m(S) = E \Lambda_D(S, S') \Omega(S') [R_m(S, S') - R_b(S)] \quad (\text{B.4})$$

$$\nu_n(S) = E \Lambda_D(S, S') \Omega(S') R_b(S) \quad (\text{B.5})$$

$$\text{where } \Omega(S') = (1 - \psi) + \psi[\phi(S')v_m(S') + \nu_n(S')],$$

$$\phi(S) = \frac{\nu_n(S)}{\theta^I - \nu_m(S)}, \quad R_m(S, S') = \frac{Z(S') + \rho\pi(\omega^*(S'))\mu q(S')}{q(S)},$$

$$Z(S) = \pi(\omega^*(S))[(1 - \rho)\varphi + \rho] + (1 - \eta)[\bar{\omega} - z_1(\omega^*(S))] \frac{p(S)\bar{h}^B}{m_{-1}}$$

$$0 = \nu_m(S)[\phi(S)n(S) - (1 - \zeta)q(S)m(S)] \quad (\text{B.6})$$

$$1 = E \Lambda_D(S, S') R_b(S) \quad (\text{B.7})$$

$$(1 - g)y = c^B(S) + c^D(S) + (1 - \bar{\omega})p(S) + \eta[\bar{\omega} - z_1(\omega^*(S))]p(S)\bar{h}^B \quad (\text{B.8})$$

The transition law of the aggregate states $S' = K(S) = [m(S), \hat{b}(S), K^E(S^{exo})]$ given the current state S and controls $\mathcal{C}(S)$ is

$$\hat{b}(S) = R_b(S)[(1 - \zeta)q(S)m(S) - n(S)] \quad (\text{B.9})$$

$$\text{where } n(S) = \psi[(Z(S) + \rho\pi(\omega^*(S))\mu q(S))(1 - \zeta)m_{-1} - \hat{b}_{-1}] + \kappa q(S)(1 - \zeta)m_{-1}$$

$$m(S) = \frac{1}{(1 - \zeta\tau^B)q(S)} \{c^B(S) + (1 - \zeta\tau^B)[Z(S) + \rho\pi(\omega^*(S))\mu q(S)]m_{-1} \\ + [1 - \bar{\omega} + \eta(\bar{\omega} - z_1(\omega^*(S)))]p(S)\bar{h}^B - (1 - g)\tau^B y\} \quad (\text{B.10})$$

B.2 Steady State

Since the downpayment and the incentive constraints are relevant ingredients in the model, I solve the steady state in the case where both constraints are binding and $\zeta \equiv 0$. The equilibrium conditions and the endogenous variables at the steady state are listed as follows.

- Prices (8): $q, p, R_b, R_m, \nu_m, \nu_n, \lambda_l^B, \phi$.
- Quantities (8): $c^B, m^B, \omega^*, Z, b^I, n^I, c^D, T$.

B.2.1 Steady State Conditions

Borrower Household (7):

$$\omega^* = [(1 - \rho)\varphi + \rho(1 + \mu q)] \frac{\theta^B}{\varphi} \quad (\text{B.11})$$

$$q - \lambda_l^B \varphi = \beta_B \pi [(1 - \rho)\varphi + \rho(1 + \mu q)] \quad (\text{B.12})$$

$$(1 - \theta^B \lambda_l^B) p = \frac{\alpha}{1 - \alpha} \frac{c^B}{h^B} + \beta_B z_1 p \quad (\text{B.13})$$

$$\varphi m^B = \theta^B p h^B \quad (\text{B.14})$$

$$Z = \pi [(1 - \rho)\varphi + \rho] + (1 - \eta)(\bar{\omega} - z_1) \frac{\varphi}{\theta^B} \quad (\text{B.15})$$

$$R_m q = Z + \rho \pi \mu q \quad (\text{B.16})$$

$$\tau^B (y - T) = c^B + [1 - \bar{\omega} + \eta(\bar{\omega} - z_1)] p h^B + (R_m - 1) q m^B \quad (\text{B.17})$$

Financial Intermediary (6):

$$n^I = (\psi R_m + \kappa)(1 - \zeta) q m^B - \psi R_b b^I \quad (\text{B.18})$$

$$(1 - \zeta) q m^B = n^I + b^I \quad (\text{B.19})$$

$$\nu_m + \nu_n = \beta_D [(1 - \psi) + \psi(\phi \nu_m + \nu_n)] R_m \quad (\text{B.20})$$

$$\nu_n = \beta_D [(1 - \psi) + \psi(\phi \nu_m + \nu_n)] R_b \quad (\text{B.21})$$

$$\phi = \frac{\nu_n}{\theta^I - \nu_m} \quad (\text{B.22})$$

$$(1 - \zeta) q m^B = \phi n^I \quad (\text{B.23})$$

Depositor Household (2):

$$1 = R_b \beta_D \quad (\text{B.24})$$

$$(1 - \tau^B)(y - T) = c^D + (1 - \bar{\omega}) p \bar{h}^D - (1 - \zeta)(R_m - 1) q m^B \quad (\text{B.25})$$

Government (1):

$$g y = T + \zeta (R_m - 1) q m^B \quad (\text{B.26})$$

B.2.2 Uniqueness of the Steady State

By (B.24), the risk-free rate is pinned down, $R_b = 1/\beta_D$. Divide (B.20) by (B.21), I solve for the MBS return R_m .

$$R_m = \left(\frac{\nu_m}{\nu_n} + 1 \right) R_b \quad (\text{B.27})$$

Combine (B.18) and (B.19) to substitute for b^I , we have a second equation of the leverage ratio $(1 - \psi R_b) n^I = [\psi(R_m - R_b) + \kappa] q m^B$, in addition to (B.23). Substituting R_m , I have the first equation of ν_m and ν_n from two expressions of ϕ .

$$\nu_m + \kappa \nu_n = (1 - \psi R_b) \theta^I \quad (\text{B.28})$$

Combining (B.21) and (B.22), we have the second equation.

$$\nu_m + \theta^I \nu_n - \frac{1}{1-\psi} \nu_m \nu_n = \theta^I \quad (\text{B.29})$$

To show the uniqueness of the steady state value (ν_m, ν_n) , take the difference of two equations to rewrite (B.28).

$$\nu_n = \frac{\psi R_b \theta^I}{(\theta^I - \kappa) - \frac{1}{1-\psi} \nu_m} \quad (\text{B.30})$$

Given certain parametric assumptions, (B.28) and (B.30) can numerically solve for a unique pair of (ν_m, ν_n) in the first quadrant.⁵³

To solve for other variables at the steady state, I first take ω^* as given (π and z_1 are thus fixed) and solve other prices and quantities. Under the condition, $Z(\omega^*)$ in (B.15) is regarded as a constant, and (B.12) and (B.16) solve for the mortgage price q and Lagrange multiplier λ_l^B , respectively.

$$\begin{aligned} q(\omega^*) &= \frac{Z(\omega^*)}{R_m - \rho\pi(\omega^*)\mu} \\ \lambda_l^B(\omega^*) &= \varphi^{-1} \{q(\omega^*) - \beta_B \pi(\omega^*) [(1-\rho)\varphi + \rho(1 + \mu q(\omega^*))]\} \end{aligned} \quad (\text{B.31})$$

(B.13), (B.14), (B.17) and (B.25) then form four linear equations in (ph^B, m^B, c^B, c^D) which can be expressed in terms of ω^* .

$$\begin{aligned} p(\omega^*) &= \frac{1}{h^B} \frac{\alpha \tau^B (1-g)y}{\alpha [\Gamma_1(\omega^*) - \tau^B \Gamma_3(\omega^*)] + (1-\alpha) \Gamma_2(\omega^*)} \\ c^B(\omega^*) &= \frac{(1-\alpha) \Gamma_2(\omega^*) \tau^B (1-g)y}{\alpha [\Gamma_1(\omega^*) - \tau^B \Gamma_3(\omega^*)] + (1-\alpha) \Gamma_2(\omega^*)} \\ m^B(\omega^*) &= \frac{\theta^B}{\varphi} p(\omega^*) h^B \\ c^D(\omega^*) &= (1-\tau^B)(1-g)y + (1-\zeta \tau^B)(R_m - 1)q(\omega^*)m^B(\omega^*) - (1-\bar{\omega})p(\omega^*)\bar{h}^D \\ \text{where } \Gamma_1(\omega^*) &= 1 - \bar{\omega} + \eta(\bar{\omega} - z_1(\omega^*)) + (R_m - 1)q(\omega^*) \frac{\theta^B}{\varphi} \\ \Gamma_2(\omega^*) &= 1 - \theta^B \lambda_l^B(\omega^*) - \beta_B z_1(\omega^*), \quad \Gamma_3(\omega^*) = \zeta(R_m - 1)q(\omega^*) \frac{\theta^B}{\varphi} \end{aligned} \quad (\text{B.32})$$

Finally, I show that there is a unique ω^* under certain distributional assumption on ω . I use (B.11) to rewrite (B.15) and (B.16).

$$\hat{q}(\omega^*) = \frac{\varphi}{\theta^B R_m} [\pi(\omega^*) \omega^* + (1-\eta)(\bar{\omega} - z_1(\omega^*))] \quad (\text{B.33})$$

(B.11) and (B.33) form a system of non-linear equations of q and ω^* , and ω^* solves the following equation.

$$\omega^* = [(1-\rho)\varphi + \rho(1 + \mu \hat{q}(\omega^*))] \frac{\theta^B}{\varphi} \quad (\text{B.34})$$

⁵³ To guarantee a unique positive solution $\nu_m, \nu_n \geq 0$, Two conditions must hold (1) $\theta^I > \kappa$, (2) $\theta^I - \kappa > \psi R_b \theta^I$. If the either condition fails, we have either $\nu_m < 0$ or $\nu_n < 0$ in the solution.

Assume ω is Pareto distributed, $\omega \sim \text{Pareto}(\underline{\omega}, \gamma)$. Differentiate the left and right hand side of (B.34).

$$\frac{dRHS(\omega^*)}{d\omega^*} = \frac{\rho\mu}{R_m}(\pi(\omega^*) - \eta\omega^*f(\omega^*)) < 1 = \frac{dLHS(\omega^*)}{d\omega^*} \quad (\text{B.35})$$

As long as $LHS(\underline{\omega}) < RHS(\underline{\omega})$ is satisfied, or

$$\underline{\omega} < \frac{[(1-\rho)\varphi + \rho]\theta^B/\varphi}{1 - \rho\mu/R_m}$$

there exists a unique interior solution to $\omega^* > \underline{\omega}$. If the mortgage is short-term ($\rho = 0$, $\mu = 1 - \varphi_0$), the inequality is reduced to $\underline{\omega} < \theta^B$. The rest of the prices and quantities are thus uniquely pinned down.

C Proof

C.1 Proof of Proposition 4.1

Proof. Equating $\bar{S}(R_m)$ and $\bar{D}(R_m)$, I express θ^B as a function of R_m which is defined as $\Theta(R_m)$. In the first case, the demand side constraint of mortgage credit is uniformly tighter than the supply side constraint, so the intermediary is the unconstrained agent that prices the mortgage backed security. Similarly, in the second case, the converse is true and the borrower's optimality condition pins down the return. In the last case in which both constraints bind, by the Intermediate Value Theorem, the supply and demand of fund pins down R_m . Note that $\hat{\phi}(R_m)$ and $\Theta(R_m)$ are strictly increasing in R_m . This guarantees that the equilibrium R_m is unique. $\square$

C.2 Proof of Proposition 4.3

Proof. By substituting $\hat{\phi}(R_m)$ using (4.1), the denominator term in (4.9) is a linear function of R_m . Under $\gamma > \gamma_0$, it is straightforward to show $\Theta^d(R_m)$ increases in R_m . The rest of the proof follows that of Proposition 4.1. $\square$

C.3 Foundation for Default Adjustment Cost

I formalize the rigidity of the default threshold in (5.2) by extending the benchmark model to include a utility cost of default. Let $\hat{\omega}^*$ and ω^* be the default threshold under the rigid and flexible case, respectively. The cost function is assumed to take the following form.

$$UC(\hat{\omega}^*; S) = s_\omega(\omega_{ss}^* - \omega^*)ph_{-1}F_\omega(\hat{\omega}^*)u_c(\tilde{c}^B, \tilde{h}^B) \quad (\text{C.1})$$

where $s_\omega \in [0, 1]$ is a constant. $u_c(\tilde{c}^B, \tilde{h}^B)$ is the marginal utility that transforms the cost in terms of borrower's consumption into utility. The borrower household uses a constant marginal utility $u_c(\tilde{c}^B, \tilde{h}^B)$ at equilibrium value to evaluate the cost. The size of the default cost consists of two parts. First, The term $ph_{-1}F_\omega(\hat{\omega}^*)$ indicates a higher utility cost of default if the house price is higher and if the default threshold

is higher (more default given distributional parameters). Second, the distance between the default threshold at the steady state ω_{ss}^* and the optimal threshold ω^* under the flexible setting determines the size of the marginal cost. This term makes mortgage default deviating from the steady state ω_{ss}^* costly compared to new borrowing in managing mortgage stock. Borrower's optimality condition with respect to $\hat{\omega}^*$ is

$$\hat{\omega}^* = s_\omega \omega_{ss}^* + (1 - s_\omega) \omega^* \quad (\text{C.2})$$

The second order sufficient condition needs to be negative to guarantee $\hat{\omega}^*$ as the unique interior maximizer.

$$S.O.C. = u_c(\tilde{c}^B, \tilde{h}^B) p h_{-1} f'_\omega(\hat{\omega}^*) \hat{\omega}^* \left[\frac{\gamma}{\gamma + 1} - \frac{s_\omega(\omega_{ss}^* - \omega^*)}{\hat{\omega}^*} \right] < 0 \quad (\text{C.3})$$

Because $f'_\omega(\hat{\omega}^*) < 0$, we need the term in the bracket to positive. This is true because

$$\frac{1}{\gamma + 1} < \frac{\omega}{\omega_{ss}^*} \leq \frac{\omega^*}{\omega_{ss}^*} \Rightarrow \frac{\gamma}{\gamma + 1} > \frac{\omega_{ss}^* - \omega^*}{\omega_{ss}^*} \geq \frac{s_\omega(\omega_{ss}^* - \omega^*)}{s_\omega \omega_{ss}^* + (1 - s_\omega) \omega^*}$$

The first inequality in the first step holds under relevant parametric values, while the second inequality in the second step holds because the ratio is increasing in s_ω .

D Calibration

Table 3: Calibrated Parameters

Parameter	Value	Internal	Target/Source
<i>Shock Process</i>			
Persistence of $\log \gamma$ process, ρ_{lg}	0.600	Y	Duration of mortgage crisis
Mean of $\log \gamma$ (normal), γ_N	4.645	Y	Mean of default rate (normal)
Mean of $\log \gamma$ (crisis), γ_C	4.430	Y	Crisis arrival rate
Std.dev. of $\log \gamma$ process, σ_{lg}	0.030	Y	Std.dev. of default rate
Share of rigid threshold, s_ω	0.950	Y	Mean of default rate (crisis)
Persistence of divertible share, $\rho_{\theta I}$	0.110	Y	Persistence of leverage
Mean divertible share (normal), θ_0^I	0.481	Y	Leverage, MBS excess return
Mean divertible share (crisis), θ_C^I	0.436	Y	Pre-crisis change of leverage
Std.dev of divertible share, $\sigma_{\theta I}$	0.037	Y	Std.dev. of leverage
Persistence of income growth, ρ_y	0.493	N	BEA, NIPA table
Mean of income growth rate, $\bar{g}_y$	0.000	N	Normalization
Std.dev. of growth shock, σ_y	0.033	N	BEA, NIPA table
<i>Financial Intermediary</i>			
Cont. prob. of intermediaries, ψ	0.75	N	Queralto (2013)
Transfer share, κ	0.0053	Y	Leverage, MBS excess return
Mortgage decay rate, μ	0.954	N	Duration of 30-year FRM, $r_{FRM} = 3.3\%$
Principal component, φ_0	0.630	N	Share of principal, $r_{FRM} = 3.3\%$
Cont. prob. of mortgage, ρ	0.85	N	Conditional prepayment rate
Max loan-to-value ratio, θ^B	0.80	N	Greenwald (2017)
Foreclosure loss rate, τ	0.30	N	Campbell et al. (2011)
<i>Household</i>			
Housing consumption share, α	0.18	N	BEA, NIPA table
Borrower's housing share, h^B	0.48	N	Landvoigt (2016)
Borrower's income share, τ^B	0.45	N	Landvoigt (2016)
Gov. expenditure share, g	0.163	N	Elenev et al. (2016)
Share of Gov. asset holding, ζ	0	N	Benchmark
Depositor's discount factor, β_D	0.988	N	Risk-free rate
Borrower's discount factor, β_B	0.904	Y	House price-to-income ratio
Risk aversion, ξ^B	1	N	Standard
Risk aversion, ξ^D	20	N	Elenev et al. (2016)

Note: the model is calibrated at annual frequency. An internally calibrated parameter means that it is not set explicitly in a closed form, but is instead chosen implicitly to match certain moment.

E Figures and Tables

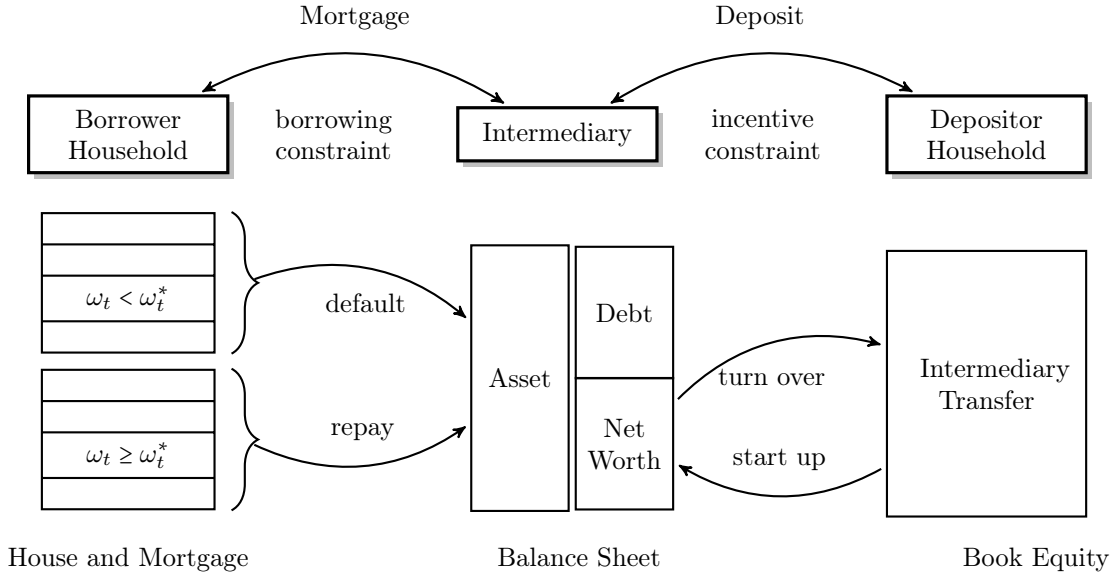


Figure 8: Graphical description of market structure, households and decisions. Each share of mortgage is tied to an equal share of house subject to idiosyncratic price risk in the borrower household. Mortgages are issued by the intermediaries who pool them as an asset or mortgage backed security. The asset is financed by either risk-free deposit or equity share held by the depositor household.

Table 4: Bank Leverage: Persistence

	(1) Unweighted	(2) Tier 1	(3) Unweighted	(4) Tier 1
L^4 .leverage	0.692*** (0.00285)	0.595*** (0.00368)	0.491*** (0.00381)	0.468*** (0.00464)
Constant	0.764*** (0.00703)	0.890*** (0.00805)	1.233*** (0.00921)	1.199*** (0.0104)
Sample	1962-2017	1962-2017	1993-2007	1993-2007
N	81,833	51,853	37,438	27,971

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Leverages are in logarithmic value. Persistence parameters are fix-effects (or within) estimators and are based on the quarterly US banks data from call reports. *Unweighted Leverage* is the inverse of equity-to-asset ratio, while *Tier 1 Leverage* refers to the inverse of risk adjusted capital ratio (Tier 1). The sample universe is the US banks with positive asset and equity values. The sample subset 1993-2007 marks the pre-crisis period when the report of risk-adjusted capital adequacy (based on 1992 criteria of 1988 Basel Accord) became available. Source: Compustat Capital IQ.

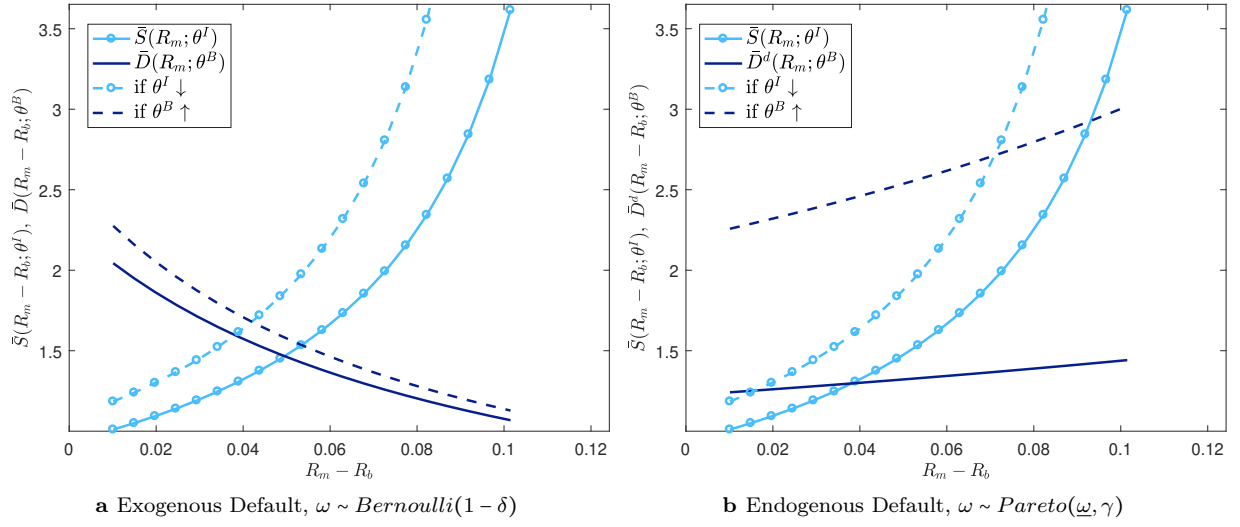


Figure 9: General equilibrium implications of the credit limits in the simplified model. $\bar{S}(R_m)$ and $\bar{D}(R_m)$ (or $\bar{D}^d(R_m)$) are endogenous limits of credit supply and demand respectively at the steady state of the simplified model in the case of exogenous (endogenous) default. The dashed lines show the impact of a 5 percent shift of θ^B and θ^I . Parameters: $\beta_B = 0.88$, $\beta_D = 0.99$, $\delta = 0.02$, $\alpha = 0.18$, $\theta^B = 0.73$, $\theta^I = 0.48$, $\tau = 0.3$, $y^D = 0.55$, $\gamma = 5$.

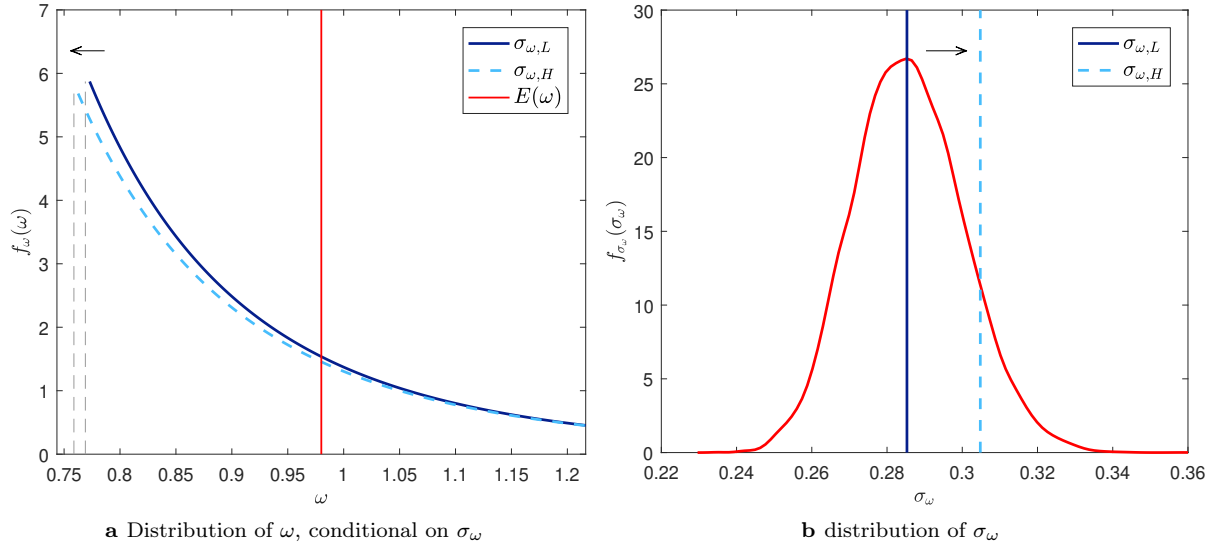


Figure 10: Panel 10a: distributions of house price risk $\omega \sim \text{Pareto}(\underline{\omega}, \gamma)$ conditional on high and low unitized risk σ_ω . The unconditional expectations of two distributions are identical. Panel 10b: kernel density of σ_ω . $\log \gamma$ is simulated 20,000 times to calculate the ergodic distribution of σ_ω , with the initial 2,000 observations dropped as burn-in. Two Parameter: $E(\omega) = 0.98$. $\sigma_{\omega,L} = 0.285$ ($\gamma_L = 4.645$, $\underline{\omega}_L = 0.784$). $\sigma_{\omega,H} = 0.305$ ($\gamma_H = 4.430$, $\underline{\omega}_H = 0.735$), $\rho_{lg} = 0.6$, $\sigma_{lg} = 0.030$.

Table 5: Bank Leverage: Summary Statistics

	Mean	SD	p10	p25	p50	p75	p90
1962-2017							
Uniweghted	2.457	0.411	2.009	2.235	2.449	2.683	2.905
Tier 1	2.188	0.332	1.808	1.996	2.180	2.376	2.593
1993-2007							
Uniweghted	2.406	0.363	1.979	2.236	2.428	2.606	2.782
Tier 1	2.235	0.333	1.833	2.041	2.236	2.449	2.646

Note: Leverages are in logarithmic value. *Unweighted Leverage* is the inverse of the equity-to-asset ratio, while *Tier 1 Leverage* refers to the inverse of risk-adjusted capital ratio (Tier 1). The sample universe is the US banks with positive asset and equity values. Source: Compustat Capital IQ.

Table 6: Bank Leverage: Growth from 2004 to 2007

	Mean	SD	p10	p25	p50	p75	p90
Unweighted							
Absolute	-0.717	8.695	-3.674	-1.738	-0.175	1.077	2.113
Relative (%)	-5.915	.	-47.224	-18.272	-1.534	7.958	13.205
Tier 1							
Absolute	0.344	1.998	-1.517	-0.527	0.462	1.302	2.228
relative (%)	3.769	.	-24.091	-6.849	5.128	12.613	18.425

Note: *Unweighted Leverage* is the inverse of the equity-to-asset ratio, while *Tier 1 Leverage* refers to the inverse of risk-adjusted capital ratio (Tier 1). The sample universe is the US banks in operation from 2004 to 2007 with positive asset and equity values. Relative growth is based on 2004 levels. Source: Compustat Capital IQ.

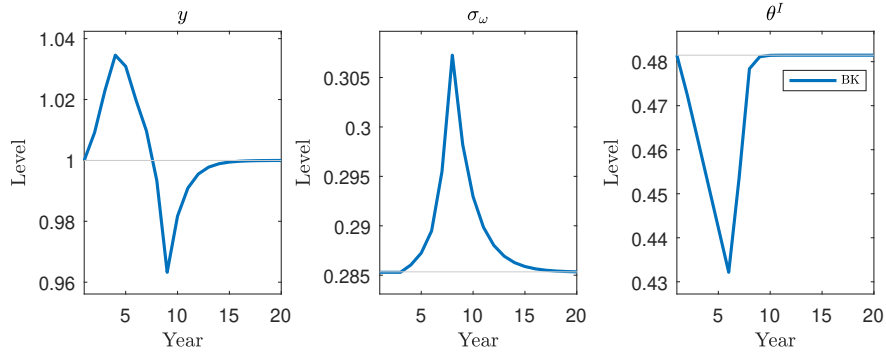
**Figure 11:** Boom-bust episode: realized path of aggregate shocks. Left Panel: income. Middle Panel: unitized risk. Right Panel: credit lending condition.

Table 7: Observed v. Simulated Moments: Mean and Standard Deviation

Variable	Log	Data		Model	
		Mean	Std.dev.	Mean	Std.dev.
<i>Price</i>					
<i>VTI</i>	Y	2.1299	0.0896	2.1931	0.1341
<i>RTI</i>	Y	0.1355	0.0046	0.1585	0.0101
<i>PRR</i>	Y	15.7302	0.8366	13.8324	1.1996
<i>Quantity</i>					
<i>PTI</i>	Y	0.0563	0.0014	0.0689	0.0042
<i>DTI</i>	Y	0.4555	0.0154	0.7296	0.0795
<i>NIS</i>	Y	0.2638	0.0354	0.2462	0.0303
<i>Pr(default)</i>	Y	0.0157	0.0500	0.0142	0.0495
<i>Leverage</i>	Y	9.3465	3.1702	10.1792	3.4251
<i>Rate/Spread</i>					
<i>R_m</i>	Y	1.0587	0.0853	1.0379	0.0416
<i>R_q</i>	Y	1.0453	0.0169	1.0331	0.0041
<i>R_b</i>	Y	1.0248	0.0129	1.0116	0.0552
<i>R_m - R_{b,-1}</i>	N	0.0398	0.0594	0.0256	0.0413
<i>R_q - R_{b,ss}</i>	N	0.0166	0.0050	0.0210	0.0041

Note: *Model summary statistics* are calculated by a simulation of 20,000 periods on an annual basis, with the initial 2,000 periods dropped as burn-in. If a variable is simulated in logarithmic form to extend the domain from positivity to the entire real line ($Log = Y$), the reported first moment is the exponentiated log-mean; the reported standard deviation of x is the average of $exp(log(x)) - exp(log(x) - SD_{lg})$ and $exp(log(x) + SD_{lg}) - exp(log(x))$, where SD_{lg} is the standard deviation derived in logarithmic form $log(x)$.

Data summary statistics are estimated with the sample before 2000, inclusive (with the exception of the default probability and the leverage due to data availability). Time series are first log-detrended for stationarity based on the augmented Dickey-Fuller tests (on stochastic and deterministic trends). The stationary components are normalized to the 2000 level, before calculating the ratios. Report of data moments are consistent with the report of the simulated counterparts.

Model Series Definition: House price-to-income ratio: $VTI = ph^B/y^B$. Rent-to-income ratio: $RTI = p_{ir}h^B/y^B$ where p_{ir} is implied rental rate. Price-to-rent ratio: $PRR = p/p_{ir}$. Payment-to-disposable income ratio: $PTI = m^B/(1-g)y^B$. Debt-to-GDP ratio: $DTI = qm^B/y$. New mortgage issuance share: $NIS = ql^B/qm^B$. Risky asset return: R_m . Implied mortgage rate: $R_q = q^{-1} + \mu$. Risk-free rate: R_b .

Data Series Definition: House price-to-income (rent-to-income) ratio is the ratio of median house price (median annual rent) to median income. Price-to-rent ratio is the ratio of median house price to median annual rent. Data are available from Census (CPS/HVS). PTI and DTI refer the ratio of mortgage payment to disposable income and the ratio of mortgage debt to GDP respectively, both available from the Board of Governor of Federal Reserves. The dollar value of the new mortgage issuance ql^B is defined as the difference of end-of-period mortgage stock and the share of old stock multiplied by decay rate μ and a constant continuation probability $\rho\pi_{ss}$. The mean of the default rate is based on the series of the foreclosure rate. The leverage refers to Tier 1 Leverage in Table 5. R_m and R_q are real MBS return and real 30-year fixed rate mortgage rate respectively. R_b is based on real 1-year treasury rate.

Table 8: Observed v. Simulated Moments: Serial Correlation

Variable	Log	Data				Model			
		1	2	3	4	1	2	3	4
<i>Price</i>									
<i>VTI</i>	Y	-0.1082	-0.1860	-0.2278	0.0719	0.8809	0.7279	0.5777	0.4477
<i>RTI</i>	Y	0.4034	0.2550	-0.1445	-0.2885	0.3266	0.0479	-0.0737	-0.1091
<i>PRR</i>	Y	-0.1269	0.3582	0.0478	-0.1173	0.4631	0.2203	0.0956	0.0376
<i>Quantity</i>									
<i>PTI</i>	Y	0.2946	-0.2003	-0.3244	0.1164	0.8809	0.7279	0.5776	0.4477
<i>DTI</i>	Y	0.5070	0.0996	0.0468	0.1526	0.8266	0.6615	0.5139	0.3932
<i>NIS</i>	Y	0.7330	0.5264	0.3989	0.2262	0.5761	0.3471	0.2087	0.1243
<i>Pr(default)</i>	Y	0.8567	0.5609	0.1965	-0.1331	0.5073	0.2479	0.1021	0.0299
<i>Leverage</i>	Y	0.5151	0.0714	-0.0053	-0.0675	0.4689	0.1989	0.0660	0.0259
<i>Rate/Spread</i>									
<i>R_m</i>	Y	-0.0076	0.1255	-0.0038	-0.0540	0.2100	0.0561	-0.0323	-0.0649
<i>R_q</i>	Y	0.3485	-0.2194	-0.1917	-0.0922	0.7010	0.5055	0.3632	0.2635
<i>R_b</i>	Y	0.1883	-0.0748	-0.0474	-0.3401	0.0818	-0.0332	-0.0648	-0.0535
<i>R_m - R_{b,-1}</i>	N	-0.1143	-0.0917	-0.0901	0.0043	-0.1824	-0.1188	-0.0757	-0.0215
<i>R_q - R_{b,ss}</i>	N	0.6363	0.2652	0.1742	0.1114	0.7009	0.5053	0.3630	0.2633

Note: Serial correlations of the model series are calculated by a simulation of 20,000 periods on an annual basis, with the initial 2,000 periods dropped as burn-in. If a variable is simulated in logarithmic form (*Log* = Y), the serial correlations are calculated using the logarithmic terms. Serial correlations of data series are estimated with the sample before 2000. Because the series of Tier 1 leverage ratio before 2000 is too short to produce reliable estimates, I use the macro leverage instead which is the ratio of total asset to total equity (data available from FFIEC). For details on variable definitions of the model and data series, see note in Table 7.

Table 9: Boom-Bust Simulation: Interest Rate Spread

$\Delta(\text{Variable})$	Boom Episode					Year (Trough)	
	Data	BK	y	σ_ω	θ^I	Data	BK
$\Delta(R_m - R_{b,-1})$	-0.0468	-0.0256	0.0025	-0.0049	-0.0243	2005	2007
$\Delta(R_q - R_{b,ss})$	-0.0013	-0.0026	-0.0026	0.0000	0.0000	2004	2007
$\Delta(\text{Variable})$	Bust Episode					Year (Peak)	
	Data	BK	y	σ_ω	θ^I	Data	BK
$\Delta(R_m - R_{b,-1})$	0.0539	0.0295	0.0119	0.0037	0.0147	2009	2009
$\Delta(R_q - R_{b,ss})$	0.0068	0.0033	0.0008	0.0022	0.0001	2008	2011

Note: $\Delta(\cdot)$ denotes demeaned variables; deviation from the unconditional mean (for data series) and deviation from the steady state (for model series) are reported in the boom and bust episodes. Besides the benchmark model (BK) in Column 2, Columns 3-5 reports the responses in alternative economies with the labeled shock at work only. Columns 6-7 reports the year when the data and model series respectively reach the trough or the peak.

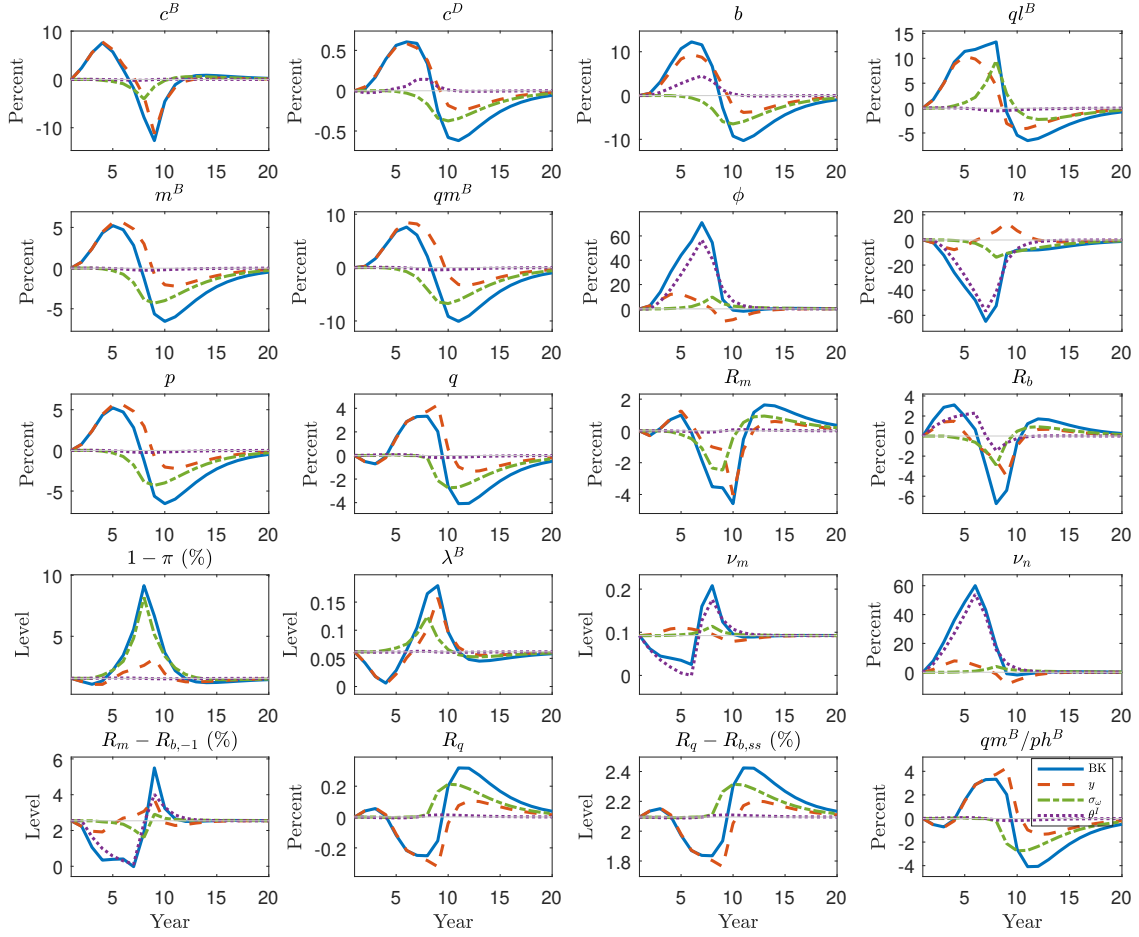


Figure 12: Response to boom-bust simulation. If the label of the vertical axis is *Level* (*Percent*), the value represents the level (the percentage deviation from the stochastic steady state) of the response variable. Paths other than the benchmark (BK) are counterfactuals where all shocks are turned off, except the one indicated in the legend.

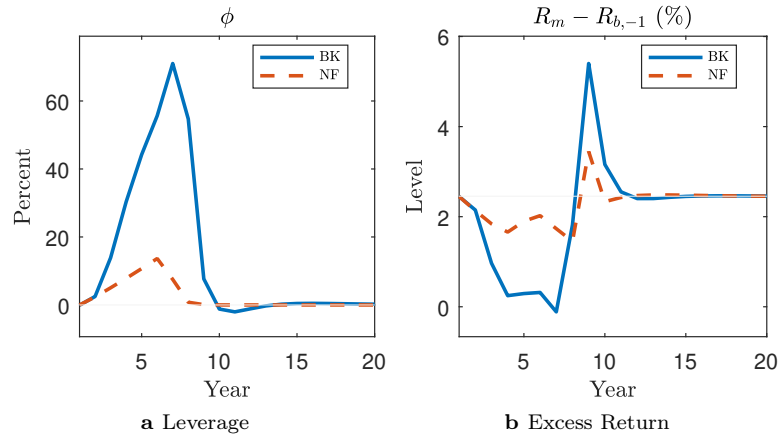


Figure 13: Boom-bust simulation with endogenous and exogenous leverage. Excess return is measured in percentage point; leverage is measured in percentage deviation from the steady state. BK = benchmark. NF = no intermediary.

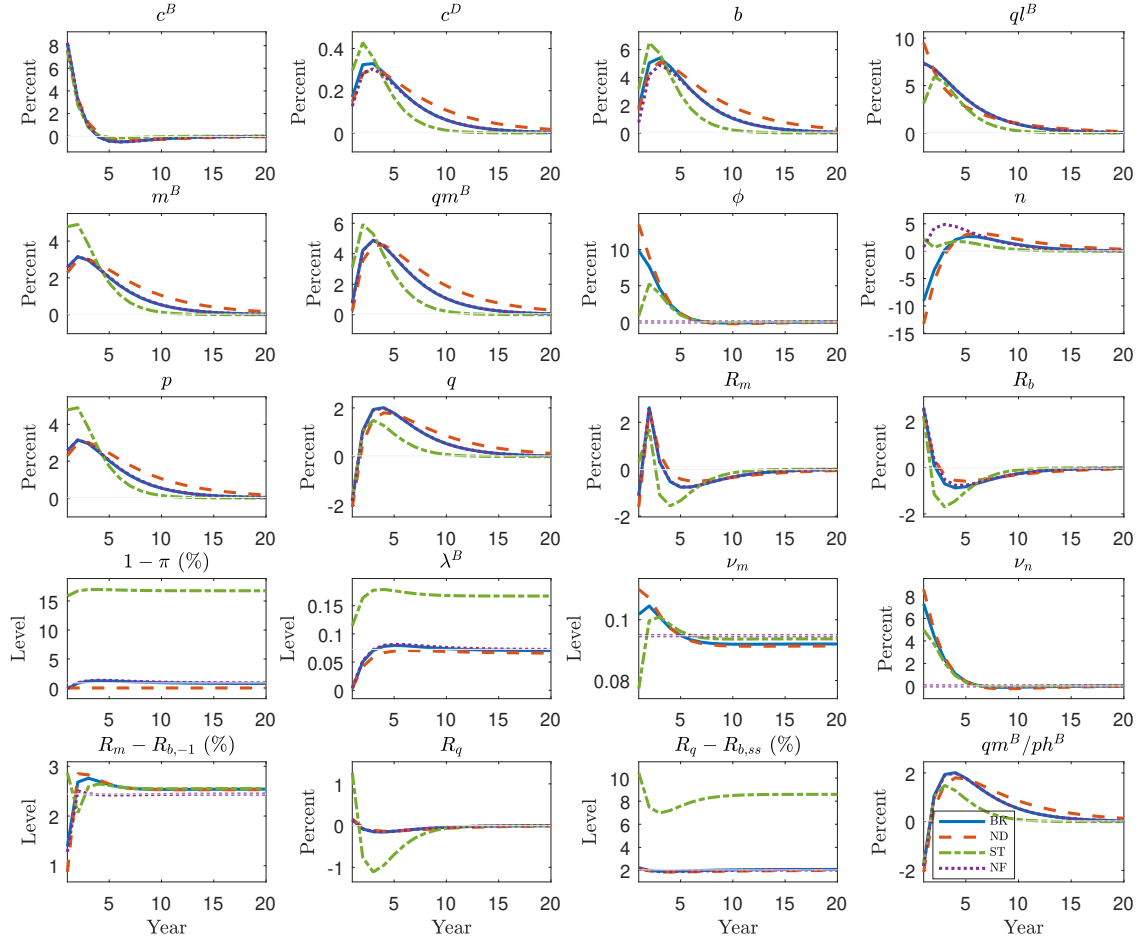


Figure 14: Generalized impulse response to +1 standard deviation of the income y . If the label of the vertical axis is *Level (Percent)*, the value represents the level (the percentage deviation from the stochastic steady state) of the response variable. BK = benchmark. ND = no default. ST = short-term. NF = no intermediary. The other lines correspond to the counterfactuals where the scenario indicated in the legend is turned on.

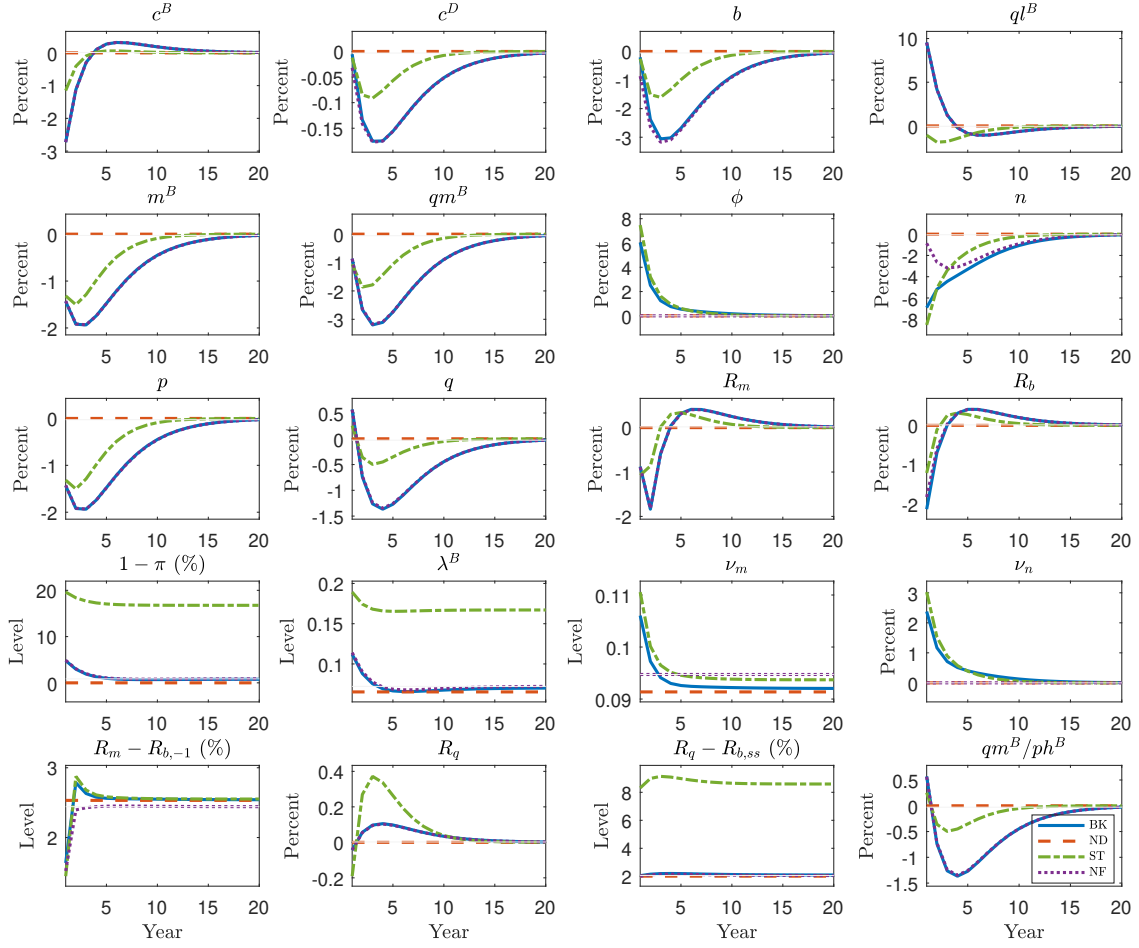


Figure 15: Generalized impulse response to +1 standard deviation of the unitized risk σ_ω . If the label of the vertical axis is *Level* (*Percent*), the value represents the level (the percentage deviation from the stochastic steady state) of the response variable. BK = benchmark. ND = no default. ST = short-term. NF = no intermediary. The other lines correspond to the counterfactuals where the scenario indicated in the legend is turned on.

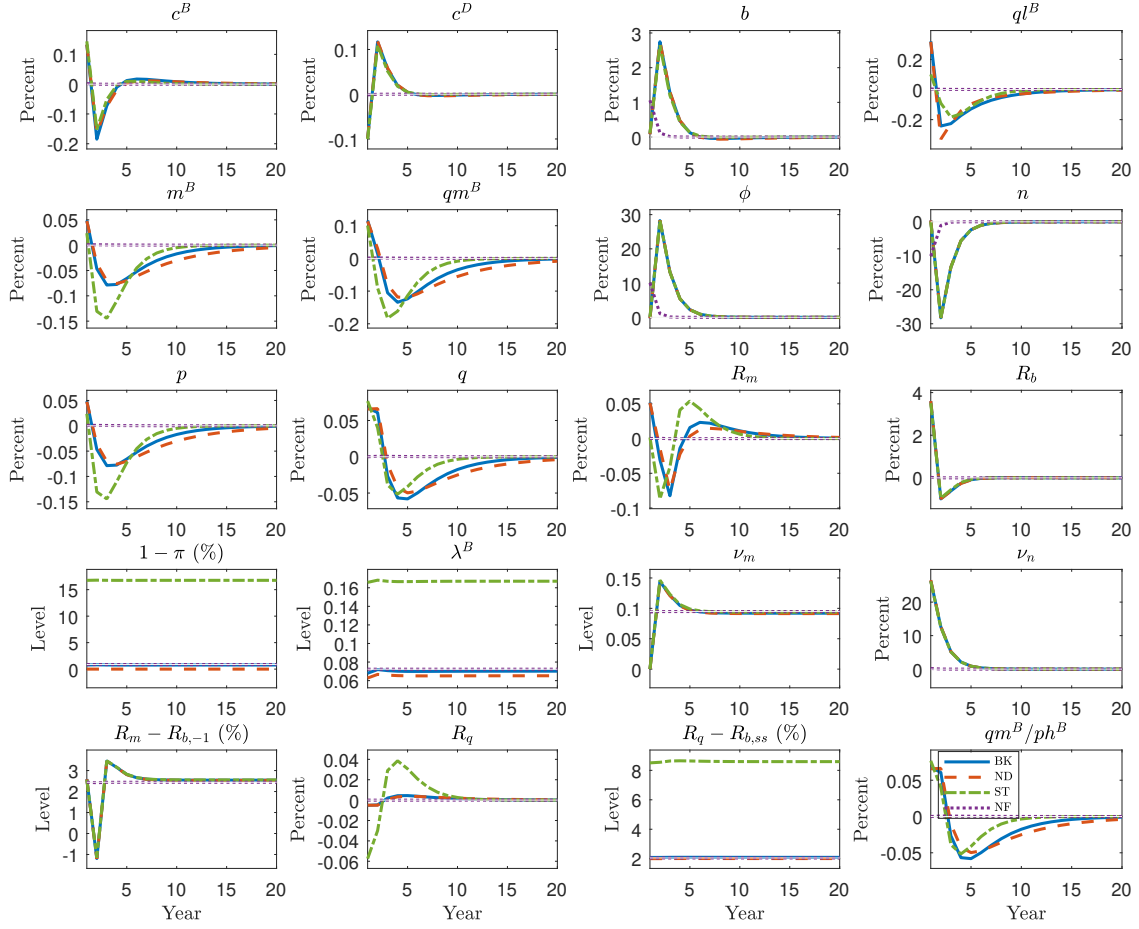


Figure 16: Generalized impulse response to -1 standard deviation of the divertible share θ^I . If the label of the vertical axis is *Level (Percent)*, the value represents the level (the percentage deviation from the stochastic steady state) of the response variable. BK = benchmark. ND = no default. ST = short-term. NF = no intermediary. The other lines correspond to the counterfactuals where the scenario indicated in the legend is turned on.

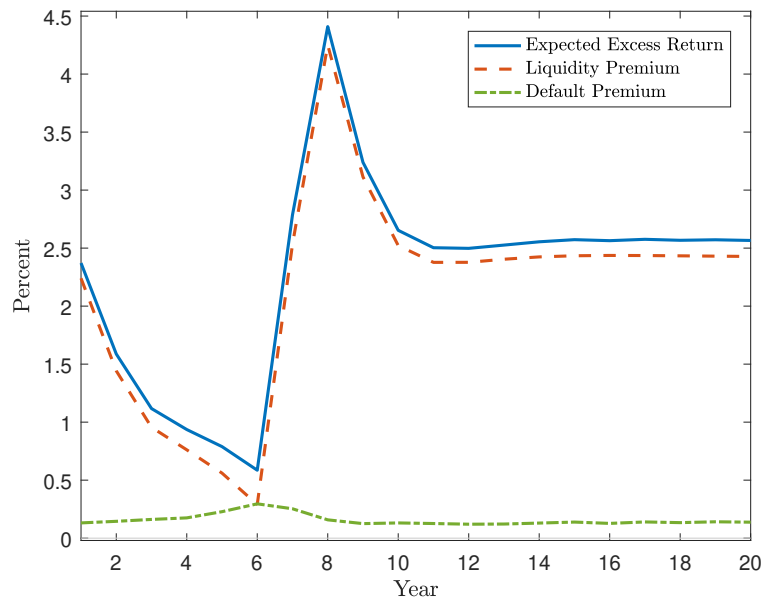


Figure 17: Decomposition of expected excess return into liquidity premium and risk premium. The simulation starts from the stochastic steady state and the exogenous processes follow the simulated path in Figure 11.